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Incentive Compatibility Since 1972

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1. Introduction

Although discussions of the role of private incentives have been included in writings on economics and political economy for more than two hundred years—at least as far back as Adam Smith's *Wealth of Nations*—the formal treatment of the subject is a recent development in economics. A seminal paper of the modern era was written in 1972 by Leo Hurwicz, a dozen years after his pathbreaking work on the foundations of decentralized resource allocation mechanisms.[1] In that paper he introduced the concept of incentive compatibility and proved that there cannot exist any informationally decentralized mechanism (or procedure) for resource allocation in private goods economies that simultaneously yields Pareto-efficient allocations and provides sufficient incentives to consumers to honestly reveal their true preferences.[2] Earlier papers (such as Vickrey 1961; Groves 1970; and Clarke 1971) formally discussed mechanisms for making resource allocation decisions in a manner compatible with individual incentives, but Hurwicz was the first to establish results for a classical, full general equilibrium model of an exchange economy. His paper was the major stimulus to the large number of papers that have subsequently appeared. For this reason we begin this retrospective at that date.

The concept of incentive compatibility, introduced by Hurwicz to capture

We would like to thank Andy Postlewaite, Roy Radner, Stefan Reichelstein, and especially John Roberts for their many helpful comments and efforts to save us from inaccuracies. Remaining errors are our own.

[1]See Hurwicz (1960) and Hurwicz (1972). The later paper continues to be an excellent introduction to the subject of incentives in resource allocation.

[2]A precise statement of this theorem is given below in Section 3.1.

the forces for individual self-interested behavior, has proven to be one of great scope, serving as an organizing principle of considerable power. Perhaps the closest analogy in economics is the concept of efficiency. For the positivist, notions of self-interested behavior lie at the foundation of all microeconomic theory. Indeed, the only outcomes that can be generally realized in any situation are those that result from individual decision makers following their own interests. For the normativist, relatives of the concept of incentive compatibility may be traced to the "invisible hand" of Adam Smith, who claimed that in following individual self-interest, the interest of society might be served. Related issues were a central concern in the "Socialist Controvery" which arose over the viability of a socialist society. It was argued by some that such societies would have to rely on individuals to follow the rules of the system. Some believed this reliance was naive; others did not. These debates led to the modern theory of mechanism design that treats incentive compatibility as a constraint on the choice of procedures able to be used to make group allocation decisions in various economic institutional contexts.

In this chapter, we present an organized overview of what is now known about the possibilities for the incentive compatible design of mechanisms. We also indicate some of the major remaining mysteries. However, incentive compatibility questions have been addressed for models of central planning, regulation of monopoly, transfer pricing, and capital budgeting, to name just a few. Therefore, rather than try to survey the entire recent literature on the subject (a book-length task), we have chosen, following Hurwicz (1972), to concentrate instead on incentives in two well-known classical general equilibrium models of resource allocation—one being the standard private goods pure exchange model, the other a simple public and private goods general equilibrium model. Thus, many papers on incentive compatibility written in the last decade will not be mentioned here. In particular we ignore the large amount of exciting work concerned with design and incentive problems in a partial equilibrium framework (see Myerson 1983 for an excellent introduction) and the work on particular institutions in which information and incentive issues are crucial (see, for example, Milgrom and Weber 1982, Wilson 1985).[3] Furthermore, even in the narrow area to which we have constrained ourselves, our survey is undoubtedly incomplete—rather, it is a personal overview of results comparing private goods economies with

[3]Special subsets of the general equilibrium environments will be mentioned in subsequent sections. These include environments restricted to quasi-linear preferences and to zero-one choices. Since many of the results known to hold in these special cases do not survive in the more general environments, we most often refer to them as examples only.

those with public goods. Two surveys from differing points of view but covering some of the same results are those of Schmeidler (1982b) and Postlewaite (1985).[4]

Our decision to concentrate on the differences between and similarities of the conclusions one may draw concerning incentive compatible design of resource allocation mechanisms in private and public goods environments allows us both to summarize a large number of contributions, many of which address this central issue, and to show how rigorous analyses of incentive compatibility have deepend and changed the conventional wisdom regarding the possibility for achieving Pareto-efficient allocations through decentralized means (such as competitive markets). That conventional wisdom before 1972, it is fair to say, could be summarized in two statements:

In classical private goods economies, Pareto-efficiency is consistent with individual self-interest since price-taking behavior is reasonable in competitive markets, especially if the number of agents is large.

In classical public goods economies, Pareto-efficiency is not consistent with individual self-interest since agents will have an incentive to "free ride" on others' provision of public goods (in order to reduce their own share of the burden of providing them).

As we show in the sections below, it is now known that these statements are seriously misleading and obscure some important and subtle distinctions between private and public goods. For the impatient reader, all of the results we detail are summarized at the end of each section of this chapter.[5] To whet the appetite, however, we briefly summarize the five main results that most effectively highlight the differences between private and public goods. The first three hold for both private and public goods environments.

(1) In classical (private *and* public goods) economies with a finite number of agents, there are *no* nonparametric mechanisms that simultaneously yield Pareto-efficient allocations and provide individual agents with incentives to report their true preferences honestly.

Thus, since agents cannot be induced to behave in an incentive compatible manner, the analysis of resource allocation mechanisms requires some prediction of agent behavior.

[4]For some other surveys of the voluminous literature on Incentive Compatible Social Choice, Implementability of Social Choice Rules, and so forth, see the surveys of Dasgupta, Hammond, and Maskin (1979), Groves (1979), and Laffont and Maskin (1982).

[5]We have delayed summaries to the end since much of the language used must be precisely defined before it is really understood. These definitions are contained in the body of the chapter.

(2) In classical (private *and* public goods) economies with a finite number of agents, there are nonparametric mechanisms that yield Pareto-efficient allocations when all agents follow their self-interest by playing a Nash-equilibrium strategy.

Since the pre-1972 conventional wisdom suggests that price-taking behavior in private goods economies with many independent agents is in each agent's interest, one might look to economies with a continuum of agents to find a difference between public and private goods.

(3) In classical (private *and* public goods) economies with a continuum of agents, there exist mechanisms that simultaneously yield Pareto-efficient allocations and provide individual agents with incentives to report their true preferences honestly. (Compare with 1 above.)

In large finite, but growing, economies, we can find a distinction between private and public goods economies for mechanism design.

(4) In classical *private goods* economies, there exist mechanisms such that the Nash-equilibrium strategy yields an "almost" Pareto-efficient allocation as the outcome and is "almost" equivalent to reporting agents' true preferences, if the economy is "large enough."

The same result does not appear to hold for public goods economies.

(5) In classical *public goods* economies, there exist mechanisms such that the Nash-equilibrium strategy is "almost" equivalent to reporting agents' true preferences, if the economy is "large enough," but it seems that none of these mechanisms simultaneously yields an "almost" Pareto-efficient allocation, no matter how many (finite number of) agents there are in the economy.[6]

We turn now to a survey of the literature that underlies these and many other facts that have been discovered in the last decade.

2. Resource Allocation Mechanisms in Classical Economic Environments

To begin our survey, we first introduce a useful model for organizing the material in this area. This model allows us to standardize notation and to compare and contrast the results of many researchers within a common framework. We hope that others, unfamiliar with this area, will also find this to be helpful.

[6]Parts of (5) remain conjecture. This is carefully discussed in Section 5.

The four primary components of our model are the environment (endowments, preferences, opportunities, etc.), the allocation mechanism (a language and an outcome rule), a reduced form description of self-interested behavior (an example is Nash equilibrium), and a concept of "good" allocations (such as Pareto-efficient, equitable, etc.). The first and the last will be familiar to economists since these components are from standard general equilibrium theory. The second and third will be familiar to game theorists since much of these components comes from standard n-person, noncooperative game theory. The analysis of incentive compatibility requires all four to be merged into a common framework which we do below.

2.1. Private Goods Model

In the classical model of a private goods economy, there are N consumers and L goods. Each consumer is endowed with an amount of each good, denoted by the L dimensional vector, w_i. We represent the consumption of the i^{th} consumer by the L dimensional vector x_i. Each consumer has a neoclassical utility function, $u_i(x_i)$, which is assumed to be strictly quasi-concave, monotonic, and C^2 on R^L_+. We assume that the consumer can only consume bundles x_i of commodities with non-negative amounts of each commodity. In some cases, we will represent the i^{th} utility function as $u(x_i,y_i)$, where y_i is the parameter defining the particular utility function from some class of functions. In the tradition of Hurwicz (1960), using the language of mechanism theory, we call $e_i = (y_i,w_i)$ the characteristic of consumer i and we call the full vector, $e = (e_1,\ldots,e_N)$, the environment.

An allocation in this classical environment is a vector of consumption bundles, $x = (x_1,\ldots,x_N)$. Several of these allocations have special significance for economists. An allocation is feasible[7] for the environment e if and only if $x_i \geq 0$ for each i and $\Sigma x_i = \Sigma w_i$. An allocation is Pareto-efficient in the environment e if it is feasible and if there is no other feasible allocation at which every consumer is at least as well off and at least one is better off. Formally, x^* is Pareto-efficient in e if and only if (i) x^* is feasible for e and (ii) if x is feasible for e, then $u_i(x_i) < u_i(x_i^*)$ for at least one i. An allocation, x, is Walrasian for e (sometimes called a competitive allocation) if (i) x is feasible and (ii) if there is a vector p in R^L_+, a price vector, such that (iia) $px_i = pw_i$ and (iib) if $u_i(x_i^*) > u_i(x_i)$, then $px_i^* > pw_i$.

[7]This definition of feasibility is standard and includes both individual feasibility and material balance. Recently, for mechanism design problems, some authors (e.g., Myerson 1981, 1983) have suggested that incentive compatibility constraints be included in the definition of feasibility to reflect the fact that certain allocations may not be attained because they require the transmission of private information, and the holders of that information may have an incentive to dissemble in transmitting it. Optimality is then defined relative to these informational/incentival constraints as a "second-best" concept. In this chapter we are interested in the possibility of designing mechanisms that yield "first-best" (sometimes called *ex post* optimal) allocations and, thus, we stay with the standard definitions.

The Fundamental Welfare Theorem applies to all environments that we have called classical (see, for example, Arrow 1951; Debreu 1959). If e satisfies the assumptions we have made, then two results hold:

If x is Walrasian in e, then x is Pareto-efficient in e.

If x is Pareto-efficient in e, then there exists a redistribution of w such that for the new environment e', x is Walrasian in e'. (A redistribution of w is a vector $w' = (w_1',\ldots,w_N')$ such that $\Sigma w_i' = \Sigma w_i$.)

It has been accepted doctrine since the time of Adam Smith (1776) that private-ownership market institutions are efficient under competitive conditions and that it is in the self-interest of the individuals to behave competitively. Stated another way, in private-ownership economies, even if all agents aggressively follow their self-interest, the market will lead them to promote the interests of the whole. The classical welfare theorems stated above provide one of the two necessary steps for a formal statement and proof of this conventional wisdom. Interpreting Pareto-efficient as "the interest of the whole," we know from these theorems that if individuals do behave competitively, they will serve this interest. The other step is the demonstration that it is in the self-interest of the consumers to behave competitively. Prior to 1972 most economists believed that fact to be either true or a good enough approximation in an economy with many consumers.

2.2. Public Goods Model

In a classical model of a public goods economy, there are N consumers, L private goods, and M public goods. Each consumer is endowed with an amount of each private good, denoted by the L dimensional vector, w_i. We represent the consumption of the i^{th} consumer by the $L + M$ dimensional vector (x_i,z). Each consumer has a neoclassical utility function, $u_i(x_i,z)$, which is assumed to be strictly quasi-concave, monotonic, and C^2 on R^{L+M}_+. We assume that the consumer can only consume bundles x_i of commodities with nonnegative amounts of each commodity. In some cases, we will represent the i^{th} utility function as $u(x_i,z,y_i)$, where y_i is the parameter defining the particular utility function from some class of functions. We assume that there is no initial endowment of public goods but that a transformation surface defines the rate at which private goods can be used to produce public goods. This surface is denoted by $T(r,z) = 0$, where r is the vector of private goods inputs. We assume for simplicity that $T(.)$ is linear; that is, if $T(r,z) = 0$, then $T(\lambda r,\lambda z) = 0$ for all $\lambda > 0$. As above, we call $e_i = (y_i,w_i)$ the characteristic of consumer i and we call the full vector, $e = (e_1,\ldots,e_N,T(.))$, the environment.

An allocation in this classical environment is a vector, $(x,z) = (x_1,\ldots,x_N,z)$. As in the case of the private goods economy, several of these allocations have special significance. An allocation is feasible for the environment e if and only if (i) $x_i \geq 0$ for each i and $z \geq 0$, (ii) $T(r,z) = 0$, and (iii) $\Sigma x_i - r = \Sigma w_i$. An allocation is Pareto-efficient in the environment e if it is feasible and if there is no other feasible allocation at which every consumer is at least well off and at least one is better off. Formally, (x^*,z^*) is Pareto-efficient in e if and only if (i) (x^*,z^*) is feasible for e and (ii) if (x,z) is feasible for e, then $u_i(x_i,z) < u_i(x_i^*,z^*)$ for at least one i. An allocation, x, is Lindahl for e if (i) (x,z) is feasible and (ii) if there is a vector p in R^L_{++}, and vectors q_i, one for each i, in R^M_+ such that (iia) $px_i + q_iz = pw_i$, (iib) if $u_i(x_i^*,z^*) > u_i(x_i,z)$, then $px_i^* + q_iz^* > pw_i^*$, and (iic) $qz - pr = 0$, where $r = \Sigma x_i - \Sigma w_i$ and $q = \Sigma q_i$.

The Fundamental Welfare Theorem applies to all public goods environments that we have called classical (see, for example, Foley 1967). If e satisfies the assumptions we have made, then two results hold:

If x is Lindahl in e, then x is Pareto-efficient in e.

If x is Pareto-efficient in e, then there exists a redistribution of w such that for the new environment e', x is Lindahl in e'.

It has been received doctrine since Samuelson (1954), that private-ownership market institutions are not efficient when there are public goods, since it is not in the self-interest of individuals to behave competitively. Stated another way, in private-ownership economies with public goods, if all agents aggressively follow their own self-interest, decentralized institutions will not lead them to promote the interests of the whole. Although it was never formalized, prior to 1972 most economists believed that in the presence of public goods, efficient allocations were impossible to attain with decentralized mechanisms if agents behaved in their own self-interest.

2.3. Allocation Mechanisms

An allocation mechanism is an abstraction of the enormous variety of institutions used to allocate resources, that is, used to choose a specific allocation given the environment. Many abstract models of allocation systems have been proposed since the seminal paper of Hurwicz (1960). We use one in this chapter that we have found to be especially useful. It does not explicitly model all the possible communication and decision relationships between every agent in the economy, nor does it explicitly model the sequences and number of iterations necessary to complete the transfer of information. We therefore refer to this as the *normal* form of a mechanism. This description is adequate initially, but we later will discuss its limitations. An allocation mechanism (in normal form), then, is simply a language and an outcome rule.

2.3.1 Language

Let M_i denote the language (message space) that agent i can use to communicate. A few specific examples of the types of messages the language might contain are a vector of proposed trades (quantity demanded or supplied), a description of i's characteristics, a list of the amounts i is willing to spend on each good, a description of i's cost structure, or a collection of conditional responses to others' proposals. Letting M be the product space $M_1 \times \ldots \times M_N$, we call M the language of the allocation process.

2.3.2 Outcome Function

The other part of an allocation mechanism is a function that associates an allocation with any vector $m = (m_1,\ldots,m_N)$ of messages from the language M. We denote this outcome function as $h: M \to A$, where A is the set of allocations to be chosen among. Many problems arise if h is not single valued. For example, agents may be unable to coordinate actions or a single agent may be unable to evaluate the consequences of his actions even if he knows the actions of others. Mechanisms that are not single valued are not well defined. (An example of such a mechanism, called the Competitive Mechanism, can be found below following Theorem 3.1.) For almost all of this chapter, we avoid these problems simply by assuming h is a function. We will point out when this is not assumed.

It is important to note that in order for this description of an outcome function to make sense, it is necessary to know something about the class of environments in which the mechanism is operating. In particular, we need to know the number of agents or consumers and the type of allocations that will be considered. The space A will look different for private goods economies than it does for public goods economies. The need for this prior information is not a handicap, but it is a limitation which should be noted.

Another point to be noted is that this formulation of an allocation mechanism is more general than might be apparent. Even though we have not explicitly modeled any form of iteration, it is possible to include mechanisms of that type in the same way that a normal form game can sometimes summarize a sequence of moves in an extensive form game. Recognition of this fact is important for the later discussion of implementation. Models of allocation mechanisms that explicitly allow for the iterative steps in a communication process can be found in Hurwicz (1960), Reiter (1974), and Smith (1979, 1982).

Finally, in some models of mechanisms in the literature, information other than m appears in the outcome functions. This "extra" information has to be viewed as common knowledge which is known to the designer of the system as well as to all the participants prior to the use of the allocation mechanism. Some examples of this can be found in the Optimal Auction literature (see, for example, Myerson 1981) where prior beliefs about which environment is the real one are allowed to be used by the outcome rule. Another type of "extra" information commonly used in the outcome function is information about initial endowments, which potentially can be audited in a way that preferences cannot, in order to ensure feasibility of the outcomes (see, for example, Postlewaite and Schmeidler 1979). Hurwicz (1972) has given the name *parametric outcome functions* to outcome rules that use other information in addition to agents' messages. These rules can be modeled as h: $E \times M \rightarrow A$ or as h: $M \times I \rightarrow A$, where I is a space of common knowledge information about the environment. We will use this formulation in Section 4.2.

2.4. *Self-interested Behavior*

In order to address issues such as those raised by Adam Smith and Samuelson concerning the performance of various ways of allocating resources in the face of self-interested behavior, it is necessary to be more precise about the particular form this behavior takes. Formally, given the allocation mechanism h, we summarize behavior as a mapping, b: $E \rightarrow M$, where the dependence of b on h is ignored only notationally. To see what is assumed in any particular b, consider a simple example, leaving others for later. We define a dominant strategy for agent i, given the class of environments E and the mechanism h, as a mapping d_i: $E_i \rightarrow M_i$ such that, for all e to be considered (all $e \in E$) and all $m \in M$, $u(h(m),y_i) \leq u(h(m/d_i(e_i)),e_i)$, where the vector $(m/s_i) \equiv (m_1,\ldots,m_{i-1},s_i,m_{i+1},\ldots,m_N)$.

A fundamental, but generally unstated, axiom of noncooperative behavior is that if an individual has a dominant strategy available, he will use it. Under this axiom, if all agents have dominant strategies d_i: $E_i \rightarrow M_i$ given h, we can let $b(e) = (d_1(e_1),\ldots,d_N(e_N))$ for all $e \in E$. Thus $b(.)$ captures the behavioral assumption that dominant strategies will be used. Of course if there are no dominant strategies for some i, the mapping is not well defined and the axiom is not sufficient to describe the behavior of the agents. In this case, it is necessary to turn to other behavioral assumptions.

When mechanism theory was originally formulated in Hurwicz (1960), the behavioral rules were more explicit and were viewed as prescriptive. For example, the rule might be *report your marginal cost*. It was assumed that agents would follow the rules. Here the behavioral rule, b, is viewed as a descriptive phenomenon, since we assume that agents will follow self-interested behavior. The function b will be our model of that behavior.

2.5. *Performance and Evaluation*

Given a description of the environment, e, of the allocation mechanism, h, and of the assumed behavior, b, we can summarize the performance of that mechanism in that environment (or class of environments) under that behavior by the mapping P: $E \rightarrow A$, where $P(e;h,b) = h(b(e))$ for all $e \in E$ is the composition of the mechanism's outcome rule and the behavioral rule. Graphically this is represented in Figure 2.1 by the commuting diagram of Reiter (1977).

In using the terminology above, we are ignoring other performance characteristics of allocation mechanisms which are important, such as the informational costs and the computational complexities. We do so in order to concentrate on the incentive aspects of mechanisms.

Once the performance of the mechanism is known, we can then compare that performance to some idealization. For example, it has been traditional to ask whether performance is consistent with Pareto-efficiency. In particular, let $S(e) = \{\text{allocations in } e \mid \text{allocation is Pareto-efficient in } e\}$. The question becomes, is $P(e) \subset S(e)$ for all $e \in E$? If the answer is yes, it is sometimes said that the mechanism implements the Pareto correspondence, S. Notice that the use of the Pareto correspondence is only illustrative. Any correspondence from E to A could be considered. Some "ideal performance functions" that have been used in the literature are (1) the Pareto-efficient

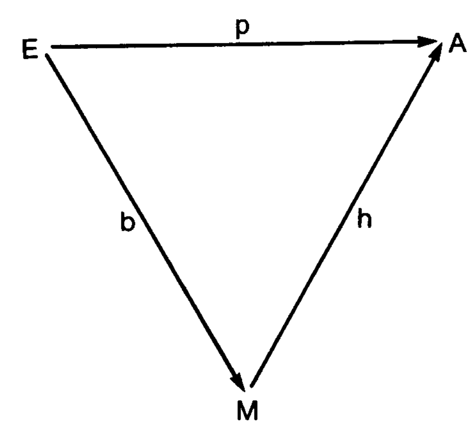

Figure 2.1. Performance rule is the composition of behavior and outcome rules.

allocations, (2) the individually rational allocations (i.e., those that leave everyone at least as well off as they were at the initial allocation), (3) the core allocations, (4) the Walrasian allocations, (5) the Lindahl allocations, (6) the allocations that yield the Shapley value, and (7) equitable allocations. We consider some of these in the sections below.

In the past there have been many variations of the basic evaluation question stated above. The original issue in the design of allocation mechanisms (see Hurwicz 1960) was the following: given a class of environments, E, and a performance criterion, P, is there a mechanism and a behavioral rule such that the performance of that mechanism under that rule is consistent with the performance criterion over that class of environments? For the purposes of this chapter it is important to note that not only the rules of the mechanism but also the behavior of the agents were to be prescribed. Hurwicz (1972) raised the incentive issue: suppose we cannot prescribe behavior but instead, as designers, must take it as given. What then can we do? In particular, given a class of environments, E, a performance criterion, S, and assumed behavior, b, does there exist a mechanism, h, such that $P(e;h,b) \subset S(e)$ for all $e \in E$? In later work, this continued to be the basic question that was asked. Sometimes it was extended to ask for a characterization of all such mechanisms; sometimes additional constraints (such as a minimal message space) were placed on the search; and sometimes the designer was allowed to use additional information (such as in the optimal auction literature). Fundamentally, however, the basic question has remained as in Hurwicz (1972).

3. Efficiency and Strong Incentive Compatibility

Partly because of the known and satisfactory efficiency properties of competitive markets and partly because of the inherent acceptability of the concept of Pareto-efficiency as a minimal welfare criteria, much of the literature on the design and evaluation of allocation mechanisms has adopted the Pareto correspondence as a primary ideal with which to compare performance. In this and the next sections, we survey the state of current knowledge about the consistency of mechanisms with efficiency under various types of behavior. In fact, one of the main, unfinished debates in this area of research is over what the appropriate behavioral assumption should be in the analysis of incentive problems.

As we indicated in Section 2 above, there is wide acceptance of the presumption that if there exist dominant strategies, then agents will adopt them. (The only possible violation occurs if agents are able and willing to collude.) With dominant strategies, then, implementation is not an issue since no agent need know anything about the others in order to choose a best message given

the information about the agent's characteristic, e_i. No sophisticated prediction of others' behavior is necessary. The only problem may be one of informational capacity or complexity of calculation, which we ignore in this paper even though in experiments in which the dominant strategy is relatively easy to calculate, many subjects still take a few iterations to find the strategy. (An analysis of such an experiment may be found in Coppinger et al. 1980 and in Cox et al. 1982.) We summarize in this section what is known about the efficiency of the performance of resource allocation mechanisms under the assumption that agents will employ dominant strategies if they exist. Later, in Section 4, we will discuss what is known for the cases when dominant strategies do not exist.

3.1. Dominant Strategies

Although mechanisms for which dominant strategies exist can easily be found, it is not easy to exhibit them if we also require the performance to be efficient. It is now known that in classical economic environments with a finite number of agents: (1) there exist mechanisms that admit dominant strategies for the agents, but (2) there do not exist (nonparametric) mechanisms that admit dominant strategies and for which the performance is consistent with the Pareto correspondence.[8] We postpone discussion of the situation in large economies in which the number of agents is infinite to Section 5.

To explain these results for finite economies, we adopt the following language. We call a mechanism h a dominant strategy mechanism on E if for all $e \in E$ and for all i there is a message $m_i'(e_i)$ such that $u(h(m/m_i'(e_i)),e_i) \geq u(h(m),e_i)$ for all $m \in M$. That is, the function $b_i(e_i) = m_i'(e_i)$, for all e_i, is a dominant strategy for i. We will call a mechanism h an efficient dominant strategy mechanism on E if it is a dominant strategy mechanism and if $P(e;h,b) \subset S(e)$ for all $e \in E$, where $S(.)$ is the Pareto correspondence.

3.2. Finite Economies and Dominant Strategy Mechanisms

The fact that there exist dominant strategy mechanisms on classical environments is easily shown. Let the set of allocations be the set of net trades. That is, in private goods environments, let consumption be $x_i = t_i + w_i$ and $A = \{t \in R^{NL} \mid \Sigma t_i = 0\}$. The trivial allocation mechanism, defined by letting

[8] Andrew Postlewaite has pointed out that if the mechanism "knows the initial endowments," then (2) is not true if parametric mechanisms are allowed. For example, let $h(.)$ be the outcome function that gives all the endowments to $i = 1$. If preferences are monotonic, then there are dominant strategies and the allocation is efficient for this dictatorial procedure. Of course, if h "does not know the endowments," then (2) is true.

M_i be any nonempty set and $h(m) = 0$ for all $m \in M$, is a dominant strategy mechanism. We call it trivial since any m_i is a dominant strategy. In public goods environments let A be the set of net trades in both private and public goods. That is, let $A = \{(t,s) \in R^{NL+M} | \Sigma t_i = 0\}$. Consumption will be $(t_i + w_i, s)$ for each i. If $h(m) = 0$ for all m, then h is a dominant strategy mechanism. Clearly these are not very desirable mechanisms. The only "good" thing about them is the existence of dominant strategies.

Nontrivial dominant strategy mechanisms do exist, however, if we restrict further the class of environments to those in which all consumers have quasi-linear utility functions. Such utility functions satisfy the condition that there is a private good, 1, say, such that $u_i(x_i) = x_{i1} + v_i(x_i \setminus x_{i1})$ where $(x_i \setminus x_{i1}) \equiv (x_{i2}, \ldots, x_{iN})$, in the private goods only model and $u_i(x_i, y) = x_{i1} + v_i(x_i \setminus x_{i1}, y)$ in the public goods model. Utility functions with this property exhibit no income effect for all goods other than good one; that is, the income elasticities of demand for all goods other than good one are zero. Since mechanisms in these environments are extensively covered in the literature (see, for example, Groves 1979), we only briefly indicate what is known to provide a background for the results for the wider class of environments considered in this chapter.

In an amazing paper which foreshadowed not only the work in incentives but also the work in incomplete information games and auctions, Vickrey (1961) discovered a particular example of a dominant strategy mechanism for classical private goods environments with quasi-linear utilities. He described his mechanism as follows: "The marketing agency might ask for the reporting of the individual demand and supply curves on the understanding that the subsequent transactions are to be determined as follows: The agency would first aggregate the reported supply and demand curves to determine the equilibrium marginal value, and apply this value to the individual demand and supply curves to determine the amounts to be supplied and purchased by the various individual buyers and sellers. The amount to be paid seller S_i would, however, somewhat exceed the amount calculated by applying this marginal value to his amount supplied; in effect for the r^{th} unit supplied, S_i would be paid an amount equal to the equilibrium price that would have resulted if S_i had restricted his supply to r units, all other purchasers behaving competitively. . . . An exactly symmetrical method could be simultaneously adopted for dealing with the demand side of the market." (Vickrey 1961, 10–12) This mechanism can be summarized easily with the help of Figure 2.2.

For Vickrey's mechanism, the messages are the "reported demand or supply" functions. The outcome rule determines the amounts of the various goods each should trade, including the "numeraire good" x_1, as follows: Agents report a demand or supply curve to the marketeer. In Figure 2.2,

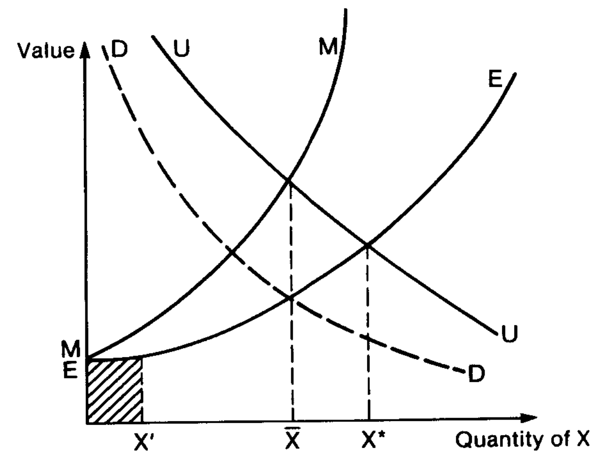

Figure 2.2. Vickrey mechanism.

the curve EE represents the "reported excess supply" of good j, say, by the agents other than buyer 1. The curve UU represents the "true marginal benefit" to 1 of an extra unit of good j in terms of the "numeraire good" 1, since we have restricted attention to quasi-linear preferences. Under the standard competitive rules for allocating resources, buyer 1 would be charged the "equilibrium" price for every unit of the good. Thus, EE represents 1's average outlay curve. To calculate buyer 1's best response to the reports of the others, 1 would calculate MM (1's marginal outlay curve), look at the intersection of MM and UU, and then send a "reported" demand function such that it intersects EE at the same allocation. DD is such a curve. As we can easily see, 1 has an incentive to understate his demand for the private good j. This problem does not arise under Vickrey's rules because, under these rules, EE is converted into the marginal outlay curve for 1 by charging 1 the area under the curve, EE, for that level of allocation. For example, the charge for x' is the crosshatched area. Since EE is now the marginal outlay curve, 1 wants an allocation, x^*, at the intersection of EE and UU. Obviously if 1 reports the demand curve UU, then 1 will obtain this allocation. Noting that UU is the appropriate response no matter where the EE curve lies, we see that UU is indeed a dominant strategy.

We can also use Figure 2.2 to describe a dominant strategy mechanism for the public goods environment. Again, buyers and sellers use, as their messages, reported demand and supply curves, although in this case these curves are usually called "willingness-to-pay functions." Since quasi-linear utility functions have been assumed, the demand and willingness-to-pay curves are defined and are the same. (With income effects this would not be true.)

EE now represents the (vertical) summation of the others' supply curves. All else remains as before, and, as before, it is in the interest of buyer 1 to report the true willingness-to-pay function UU. Groves (1970 and 1973) later discovered the general class of these mechanisms, and in the public goods environment the Vickrey mechanism becomes the Demand Revelation Mechanism independently discovered by Clarke (1971) and Groves and Loeb (1975).

As with the trivial mechanisms, the allocations produced by the Vickrey mechanism are rarely fully efficient. As Vickrey (1961, 13) observed: "The basic drawback to this scheme is, of course, that the marketing agency will be required to make payments to suppliers in an amount that exceeds, in the aggregate, the receipts from purchasers." Clarke and Groves/Loeb also noted this but were able to adjust the rules so that a surplus would be generated each time. In both the private and public goods cases, the "right" (i.e., Pareto-efficient) level of all nonnumeraire commodities would be chosen. However, not all of the numeraire good would be allocated. Thus the allocation would not be fully Pareto-efficient.

Although it appears that these Demand Revealing Mechanisms may be better than the trivial mechanisms, neither satisfies the criterion of Pareto-efficiency. Vickrey (1961, 13), aware of this problem went on to remark: "It is tempting to try to modify this scheme in various ways that would reduce or eliminate this cost of operation while still preserving the tendency to optimum allocation of resources. However, it seems that all modifications that do diminish the cost of the scheme either imply the use of some external information as to the true equilibrium price or reintroduce a direct incentive for misrepresentation of the marginal-cost or marginal-value curves." With the advantage of hindsight, we can rephrase this as follows: there do not seem to be any mechanisms that do not use specific prior information, that produce efficient allocations, and that provide the "appropriate" incentives to reveal correct information. In 1972 Hurwicz formalized and proved this now well-known fact for classical private goods economies. We turn to these results now.

3.3. Finite Economies and Efficient Dominant Strategy Mechanisms

In his 1972 paper, Hurwicz considered whether informational decentralization, Pareto-satisfactoriness, and individual incentive compatibility could be combined simultaneously in one mechanism. Informational decentralization was formalized as requiring (1) a nonparametric outcome function and (2) a behavioral rule which depended, for each agent, only on the agent's own characteristic. As stated by Hurwicz (1972, 326), "the requirement of informational decentralization enters through the postulate of 'privacy,' which means that no participant, including an enforcement agency if any, has any direct knowledge of others' preferences, endowments, technologies, etc., except possibly the restriction to the *a priori* given classes E_i." Individual incentive compatibility was conceptualized by Hurwicz (1972, 323) as "no one should find it profitable to 'cheat,' where cheating is defined as behavior that can be made to look 'legal' by a misrepresentation of the participant's preferences or endowment, with the proviso that fictitious preferences should be within certain 'plausible' limits." It later became apparent to researchers in this area that the appropriate formalization of this concept was the requirement that the mechanism be a dominant strategy mechanism. To see this we consider the following more formal model.

In the Hurwicz model an allocation mechanism is an outcome rule and a prescription of behavior in the form of specified "response functions," $f_i(m^*;e_i) = m_i$ for each i. These rules instruct each agent which message, m_i, to send in response to the "previous" joint message, m^*, of all the others. An outcome is then determined by first looking at "equilibria" of f: that is, a joint message m is an equilibrium for e if and only if $f_i(m;e_i) = m_i$ for all i. Next, the outcome function $g(.)$ is applied to the equilibrium message m to yield the final allocation: that is, the outcome is $g(m)$ where m is an equilibrium for e. Let $c(e;f)$ be the equilibria for e under the response rules, f. Then the result of the mechanism, if all follow the rules, are the allocations $a = g(c(e;f))$. If all agents act as if their characteristics are e_i^*, then the outcome is $g(c(e^*;f)) = a^*$. If we let $M = E$ and $h(m) = g(c(e;f))$, then h is an allocation mechanism (as defined above in Section 2) that yields the same allocations as the Hurwicz formulation. In this form, with characteristics as messages, these mechanisms are called *Direct Revelation Mechanisms*.[9]

We can now formalize the original Hurwicz concept of individual incentive compatibility as follows: the mechanism given by (f,g) is individually incentive compatible for the class of environments E if and only if, for all i and all e and all $e_i^* \in E_i$, $u(h(e),e_i) \geq u(h(e/e_i^*),e_i)$, where $h(e) = g(c(e;f))$ for all $e \in E$. This requires that e_i be a dominant strategy for i when e_i is i's characteristic. As it stands this does not seem to require that h be a dominant strategy mechanism, but it was soon noticed that (f,g) is individually incentive compatible for E if and only if h is a dominant strategy mechanism for E. It was a short step from this observation to the recognition

[9] Some authors have used the term *Direct Revelation Mechanism* to refer to mechanisms for which messages are characteristic *and* where reporting the true e_i is a dominant strategy. We believe that the form of the mechanism (Direct Revelation) should be kept separate from its incentive properties and, therefore, that if reporting e_i is a dominant strategy, then (h,E) is an *Incentive Compatible Direct Revelation Mechanism*.

that if h': $M \rightarrow A$ is a dominant strategy mechanism for E, then there is a direct revelation version of h' which is also a dominant strategy mechanism. This insight, of Gibbard's (1973), has been codified as the Revelation Principle (see Harris and Raviv 1979; Myerson 1979) and is straightforward to prove. For some possible drawbacks see Postlewaite and Schmeidler (1986) and Repullo (1983). Putting these results together we see that there is an individually incentive compatible mechanism for E if and only if there is a corresponding dominant strategy mechanism for E. With these formalizations, we can now turn to the key result.

THEOREM 3.1 (Hurwicz 1972). If E is the classical private goods environments with at least two agents, there is no efficient, dominant strategy, nonparametric mechanism such that $u(h(b(e),e_i) \geq u(w_i,e_i)$ for all i and for all $e \in E$.

PROOF. See Hurwicz (1972).

The last condition in the theorem, which has come to be called *individual rationality,* requires that the mechanism allow each participant a no-trade option. One particularly interesting example of a mechanism satisfying this condition is the Competitive Mechanism, which can be defined as a direct revelation mechanism as follows. (There are other possible representations, but this is the easiest one with which to work.) The message of any agent i is that agent's characteristic, and the outcome function picks net trades. Thus, h: $M \rightarrow A$ is defined as: Given a characteristic e_i, let $D(p,e_i) = \{x \in R^L \mid u(x' + w_i,e_i) > u(x + w_i,e_i) \Rightarrow px' > px = pw_i\}$ be the demand correspondence for agent i with characteristic e_i where $p \in R^L_+$, the space of all prices p. Let $C(e) = \{p \in R^L_+ \mid \Sigma D(p,e_i) = 0\}$ denote the set of competitive equilibrium prices for the environment e. Then, h assigns the net trade $D(C(e),e_i)$ to the agent with the reported characteristic e_i. Now, since the competitive mechanism is also efficient, by Hurwicz's theorem it cannot be a dominant strategy mechanism; that is, it is not individually incentive compatible in the sense that all agents have an incentive to correctly report their true characteristics.

This theorem of Hurwicz (1972) thus provided a formal proof of the Vickrey hunch and, simultaneously, established that a search to find an efficient dominant strategy mechanism, which was also individually rational, was doomed to failure. Left undecided was whether removal of the requirement of individual rationality would allow discovery of an efficient, dominant strategy mechanism. Theorem 3.4 below resolves this question negatively. Also left open was the subset of E for which incentives and efficiency were incompatible. This was partially answered in Ledyard (1977) as "almost all of E."

Even though the Hurwicz impossibility theorem established that the conventional wisdom for classical private goods environments was incorrect if there existed a finite number of agents, few were surprised to find that his result was also valid for classical public goods environments. In Ledyard and Roberts (1974), a diagram used by Malinvaud (1971), who attributed it to Kolm, was adopted and with a modification of the Hurwicz proof the following theorem was shown.

THEOREM 3.2 (Ledyard and Roberts 1974). If E is the set of classical public goods environments with at least two agents, there is no efficient, dominant strategy, nonparametric mechanism such that $u(h(b(e)),e_i) \geq u(w_i,0,e_i)$ for all i and for all $e \in E$.

PROOF. We have included a proof of this theorem in the appendix to this section since the Ledyard and Roberts (1974) paper is relatively inaccessible.

Again this left open the question of the existence of an efficient, dominant strategy mechanism if individual rationality were not required, but this gap was soon filled. Hurwicz (1975) showed that when the number of agents is at least three, there is no mechanism with a "smooth" outcome function h that both is efficient and admits a dominant strategy.

A somewhat indirect, but, in the end, more wide-ranging theorem was obtained in a sequence of papers dealing with the class of Groves mechanisms described earlier. First, Green and Laffont (1977) established that if utilities are restricted to be quasi-linear but allowed to be nonconcave, then the only dominant strategy mechanisms that choose an efficient level of the public good are Groves mechanisms. Walker (1978) demonstrated that even if utilities are restricted to the class of concave, quasi-linear functions, this characterization remains valid. Finally, Green and Laffont (1978) and Walker (1980) showed that there is no Groves mechanism which "balances the transfers" over the whole class E', the subset of classical public goods environments with quasi-linear utility functions. (A mechanism is said to "balance the transfers" if the final allocations produced by the mechanism satisfy the balance condition $\Sigma x_i + r = \Sigma w_i$.) Since balanced transfers are a necessary condition for efficiency, this collection of papers (see also Holmstrom 1979, Makowski and Ostroy 1984b) established the following theorem.

THEOREM 3.3. If E is the space of classical public goods environments with at least two agents, there is no efficient, dominant strategy, nonparametric mechanism.

PROOF. Follows from Green and Laffont (1977, 1978), Walker (1980), and Holmstrom (1979).

Finally, a unifying result has been established by Hurwicz and Walker (1983) for all classical economies, both private and public. In fact, they went even further and proved that the failure of existence of efficient dominant strategy mechanisms is "generic" on a large set of classical economies with quasi-linear preferences and more than two agents.

3.4. Summary

Combining all the results in the previous sections, we find it relatively easy to summarize the state of knowledge concerning efficient, incentive compatible mechanisms in Theorem 3.4.

THEOREM 3.4. In classical environments, both private and public, with a finite set of agents greater than one, there exist nonparametric, dominant strategy mechanisms but, there do not exist nonparametric, efficient, dominant strategy mechanisms.

The net effect of the research in this area has been to verify Hurwicz's conjecture (which we first heard in 1967) that informational decentralization, welfare maximization, and incentive compatibility are unattainable simultaneously.

Appendix: Proof of Theorem 3.2

Following is a slightly modified version of the proof in Ledyard and Roberts (1974).

The economy we construct has two identical consumers, one private good, x, and one public good, z, that can be produced from the private good under constant returns to scale. By a choice of units, the transformation of private into public good is one-for-one, that is, the production relation $g(*)$ is given by $z = g(x) = x$. Each consumer holds one unit of private good and has preferences that are given by the indifference map in Figure 2.3. For $z < x$, the indifference curves have slope of -1, whereas for $z > x$, the slopes are -3.

It is convenient to represent this economy graphically (Figure 2.4) by means of an analogue of the Edgeworth box diagram. This construction was

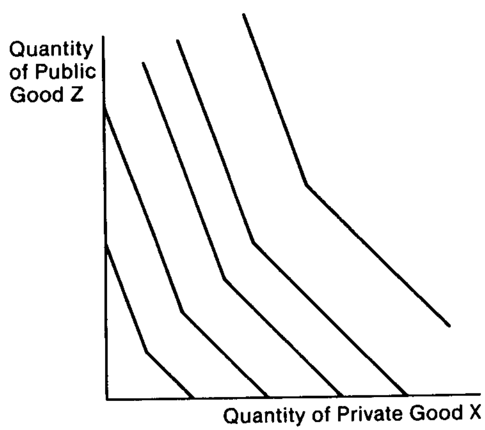

Figure 2.3. Consumer preferences for public and private goods.

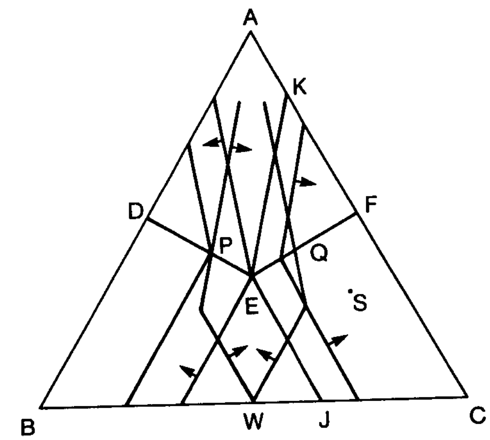

Figure 2.4. Kolm triangle showing efficient allocations with one public and one private good.

used by Malinvaud (1971), who attributed it to Kolm. The equilateral triangle in Figure 2.4 has height 2. Since the sum of the distances from any point in the triangle to the three sides is a constant, and since a feasible allocation (x_1,x_2,z) in this economy satisfies $x_1 + x_2 + z = 2 = w_1 + w_2$, a one-to-one correspondence exists between points in the triangle and the feasible allocations: using the point B as the origin for the first agent and C as that for the second, we see that point S corresponds to an allocation where z is the distance from S to BC, x_1 is the distance from S to AB, and x_2 is the distance from S to AC. The initial position (1,1,0) is then the point W on BC. Sample indifference curves for the two agents are shown. Pareto optima correspond to "double tangencies," and thus the Pareto optima are the points along DEF.

The points on PEQ are the Pareto optima in this economy that are preferred or indifferent for each agent to the initial allocation, W. We refer to the set of Pareto optima that are individually rational as the contract curve.

Any mechanism that selects allocations on the contract curve must select some point on PEQ if the agents reveal their true preferences. Suppose that outcome were on the segment PE. Then, if the second consumer reveals his true preferences, the first agent will be better off if she can, by misrepresenting her preferences, shift the apparent contract curve into the region to the right of JEK.

Clearly the agent can do this. For example, she can use the strategy that can be rationalized as being the true response of an agent with preferences given by straight line indifference curves with slope -3. This is illustrated

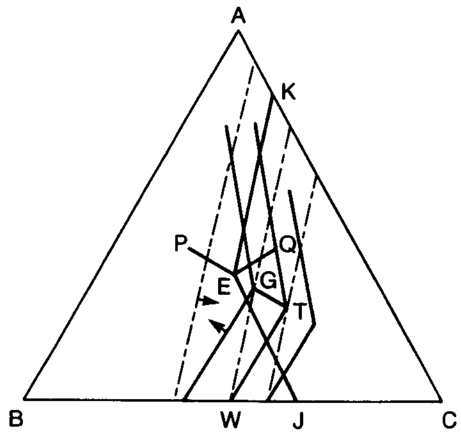

Figure 2.5. Example demonstrating that agent 1 can benefit by distorting reported preferences.

in Figure 2.5, where the apparent contract curve is now *GT*. Since the final allocation must be on *GT*, it is not individually incentive compatible for the first agent to reveal her true preferences (i.e., the strategies of telling the truth, m_i^*, do not constitute a Nash equilibrium). This result is, of course, what one would have expected: it ought not to be any easier to obtain incentive compatibility with public goods than in their absence—the case examined in Hurwicz (1972).

4. Efficiency Without Dominant Strategies

In this section, we retain the requirement that the mechanism's performance be consistent with the Pareto correspondence. But we must give up the requirement that there exist dominant strategies. This immediately opens up the question of which of the many other equilibrium concepts should be used as a behavioral rule. This is really an empirical question since, in designing a mechanism h, we must predict how a group of N individuals with characteristics e_i, $i = 1, \ldots, N$, will behave when confronted with the mechanism. Which (equilibrium) message will result when h is implemented?[10]

[10]In the incentive literature the word *implement* has come to mean that it is possible to match some desired *performance rule*, $P: E \rightarrow A$, with a mechanism $h: M \rightarrow A$ *under some behavior* $b: E \rightarrow M$. That is, h is said to *b-implement P on E* if $h[b(e)] \subseteq P(e)$ for all $e \in E$. When we speak of implementing a mechanism, we mean actually using the mechanism to determine some allocation.

To answer this question we need to know more about the mechanism than its normal form. We need to know, for example, how many iterations of information transmission are allowed. What is the stopping rule? Is communication through a central "computer," is contact random, or must we search out information? As those familiar with experiments will point out, the outcome function alone is insufficient to describe an institution as it might be actually implemented.

Two extreme examples will illustrate this point. One conceivable implementation of a mechanism, $h: M \rightarrow A$, is as a sealed bid auction, a one-iteration process. Under this implementation, each agent is required to send m_i without knowledge of the others' messages. Upon receipt of all m_i, the "auctioneer" announces the allocation, $h(m)$. A second possible implementation of the same mechanism, h, is as an iterative procedure with an endogenously determined number of iterations. In this implementation, each agent sends m_i without knowledge of the others' messages. If, for every i, m_i exactly matches the previous m_i, the process stops and the "auctioneer" announces $h(m)$. If, for at least one i, m_i is different from i's previous message, then another iteration occurs. Obviously we can also conceive of innumerable other implementations where the stopping rule for the iterative procedure is to stop after T iterations unless all m_i match the previous messages at some prior iteration. (See Smith 1977, 1979, and 1982 for some examples and a discussion.) Although each of these extensive forms of the mechanism, h, may be represented by the same normal form, it seems unlikely that agents' behavior will be the same under each form. That is, even if e were the same, we would expect the final allocation to be different for a one-iteration process than for an endogenous iteration process.

Although it is an unsettled empirical issue how agents will behave in each case, we can still point to several models that are adequate as first attempts to explore the issue. We suggest that the "model" best suited to analyze the one-iteration implementation is the one common in modeling auctions—the incomplete information-game model with common knowledge and a Nash (Bayes) equilibrium as the behavior rule. This model has the additional advantage of being normatively pleasing in that (Bayesian) agents should play this way (see Myerson 1983). For the endogenous iteration model, the natural normative choice of behavior (how the agents *should* behave) would be a Nash (Bayes) equilibrium of the repeated, incomplete information game with the number of stages endogenous. Characterization of these games remains an unsolved problem and thus, in place of that natural choice, we turn to the Nash equilibrium of the "complete information" game. We do not suggest that each agent knows all of e when he computes m_i, just as in real markets no auctioneer knows the excess demand function when equilibrium prices are calculated. We do suggest, however, that the complete information Nash game-theoretic equilibrium messages may be the possible equilibrium of the iterative process—that is, the stationary messages—just as the demand-equal-supply price is thought of as the equilibrium of some unspecified market dynamic process. We have misgivings with each of these models as representations of actual behavior, although there is some evidence that each may be reasonably accurate (Smith 1979 discusses some experimental evidence).

4.1. Nonparametric Mechanisms

Nonparametric mechanisms are those for which the message space M is the same for all environments $e \in E$ and the outcome function, $h: M \to A$, is a function of the joint message m only; that is, those in which the designer cannot incorporate any information other than that received from the agents (see Hurwicz 1972, 310). A nonparametric mechanism is said to be efficient on E under the behavior, b, if $P(e) = h(b(e;h))$ is Pareto-efficient in e for all $e \in E$. In this section we explore the existence and the characterization of nonparametric mechanisms that are efficient on classical environments under various types of behavior.

4.1.1. Bayes Equilibrium

As indicated above, if a mechanism is implemented as a one-iteration sealed-bid auction, a reasonable candidate for the description of behavior is Bayes-equilibrium behavior based on some common knowledge prior beliefs. A precise formulation of this behavioral postulate follows.

Given the class of classical environments E, consider a given nonparametric mechanism h. The Bayesian behavioral rule is specified by first assuming that each agent has a prior density on E, say $q_i(e)$. The vector of priors $q = (q_1, \ldots q_N)$ is assumed to be common knowledge. Letting $d_i: E_i \to M_i$ denote a decision rule for i, the vector of decision rules $d = (d_1, \ldots, d_N)$ is called a Bayes-equilibrium if and only if, for each i and for each $e_i \in E_i$,

$$\int_{E_{\sim i}} u(d(e);e_i)q_i(e \mid e_i)de_{\sim i} \geq \int_{E_{\sim i}} u(d(e)/m_i;e_i)q_i(e \mid e_i)de \text{ for all } m_i \in M_i,$$

where $E_{\sim i} \equiv \Pi_{j \neq i} E_j$. The result, based on the Revelation Principle, is that if d is a Bayes equilibrium for the mechanism h, then another direct revelation mechanism, $F: M \to A$, can be defined where $M_i = E_i$ for each i and $F(e) = h(d(e))$ such that the identity map $I(e) = e$ is then a Bayes equilibrium for the mechanism F. Thus, if it is possible to find an efficient nonparametric mechanism under Bayes-equilibrium behavior, then it must be possible to find a direct revelation, efficient nonparametric mechanism under Bayes-equilibrium behavior.

We can now ask whether there are efficient, nonparametric mechanisms for classical environments under the Bayes-equilibrium behavioral assumption. The answer is basically no as can be seen from the following theorem.

THEOREM 4.1 (Ledyard 1978, 1979). Given any vector of priors q, the direct revelation mechanism h has the identity map, $I: E \to E$, as a Bayes equilibrium if and only if h is a dominant strategy mechanism in e for almost every $e \in E$ with respect to q.

PROOF. See Ledyard (1978).

Thus, if h were an efficient nonparametric mechanism on E under Bayes behavior, there would be an efficient direct revelation mechanism, F, with the identity map $I(.)$ as a Bayes equilibrium. This in turn implies that F is an efficient dominant strategy mechanism for almost all of E. But by the results of Section 3, this is impossible for classical environments. Therefore, there can be no efficient, nonparametric mechanisms for classical environments under Bayes-equilibrium behavior. The agents' use of the additional information on the prior distribution, q, does not help. Two facts should help in understanding this result. Requiring $h(d(e))$ to be efficient for all e (a form of *ex post* efficiency) is much stronger than requiring *ex ante* efficiency in expected utility, as is usually done in the optimal auction literature. Also, in this theorem h is not allowed to depend on q as is customarily the case in that literature. (We analyze that case later in Section 4.2.) Therefore, the impossibility result should not be too surprising.

To summarize, we state:

THEOREM 4.2. In classical environments with a finite set of agents, there are no efficient, Bayes-equilibrium, nonparametric mechanisms.

PROOF. Follows from the theorems of Section 3 and Theorem 4.1.

4.1.2. Nash Equilibrium

Having not had much luck with one-iteration implementations of mechanisms, we consider next an idealization of the behavior expected in an infinite-iteration implementation of a mechanism. Given a mechanism defined by the language E and outcome function h, we define the Nash behavioral rule $b^N: E \to M$ as follows: For all $e \in E$, and all i

$$u(h(b^N(e));e_i) \geq u(h(b^N(e)/m_i);e_i) \text{ for all } m_i \in M_i.$$

As with Bayes equilibria, there can be a problem of too many equilibria; however, this will not be an issue in our analysis.

Now we can ask the main question: Are there any efficient, nonparametric mechanisms on classical environments under Nash behavior? The answer is yes. In addition, there are several results that characterize a wide class of such mechanisms. First we discuss five specific mechanisms; then we turn to the characterizations.

In the face of the pessimism expressed in the literature in the search for efficient, incentive-sensitive mechanisms in the early seventies, we were somewhat surprised to discover a mechanism to allocate public goods in classical environments whose Nash equilibria were Pareto-efficient (Groves and Ledyard 1977). Soon many more such mechanisms were found.

Private Goods Environments. Many of the mechanisms discovered to date that are efficient under Nash behavior (in private goods environments) have the additional property that they select Walrasian allocations. Much of the work in this area has been summarized by Schmeidler (1982b), who also provided one of the first examples (Schmeidler 1980) of a mechanism whose Nash-equilibrium allocations are Walrasian in classical environments and are, therefore, efficient. A slightly later version has the additional desirable property that its Nash equilibria are also strong Nash equilibria. This mechanism is described as follows. The message space is given by

$$M_i = \left\{ (p \cdot t) \in R^L_{++} \times R^L \mid p \cdot t = 0 \text{ and } \sum_i p_i = 1 \right\} \text{ and } M = \prod_i M_i.$$

The outcome rule, $h = (h_1, \ldots, h_N)$, is then defined for each $m = (m_1, \ldots, m_N)$ by

$$T_i = \{k \mid p_k = p_i\} \text{ and } h_i(m) = t_i - \sum_{k \in T_i} \frac{t_k}{\# T_i} \text{ for all } i.$$

THEOREM 4.3 (Schmeidler 1980). In classical environments with initial endowments, w_i, that are positive in each coordinate, with utility functions that have continuous partial derivatives, and with at least three agents, $N \geq 3$, (i) every Nash equilibrium is a strong equilibrium and (ii) the set of Nash-equilibrium allocations is the set of Walrasian allocations.

PROOF. See Schmeidler (1980).

One problem with the Schmeidler mechanism, however, is that the outcome rule is not continuous. Thus small variations in messages can cause large jumps in the allocation. If only Nash equilibria were assumed to be implemented, this would not cause difficulties; however, as we indicated above, Nash equilibria are plausible as a model of the probable outcomes only if a number of iterations occur. Since we would expect to see terminal messages close (but not necessarily equal) to Nash equilibria, any discontinuity in the outcome rule, especially near Nash equilibria, means it is difficult to approximate the eventual outcome, even though the messages were almost "right." If the outcome rule were continuous, we would know that if the messages are close to Nash equilibria, then the allocations will be close to Nash-equilibrium allocations. Because of the fallibility of information transmission, it is highly desirable to have outcome functions that are continuous.

Hurwicz (1979a) has exhibited an allocation mechanism with the desired features. Let the message space be given by: $M_i = \{(p,t) \in R^{L-1}_{++} \times R^1 \times R^{L-1} \mid p \gg 0\}$. The outcome function is then defined by: $h(m) = (h_1(m), \ldots, h_N(m))$ where $h_i(m) = (r_i(m), s_i(m))$ and $s_i(m) = t_i - t_{\sim i}$, where $t_{\sim i} \equiv \{\Sigma_{j \neq i} t_j\}/(N - 1)$, and $r_i(m) = -p_{\sim i} s_i(m) - L_i(m) + S_i(m)$, where $p_{\sim i} \equiv \{\Sigma_{j \neq i} p_j\}/(N - 1)$, $L_i(m) = (p_i - p_{\sim i})^2$, and $S_i(m) = p_{\sim i} t_{\sim i} + p^2_{\sim i} + \{\Sigma_{j \neq i} p^2_{\sim j}\}/(N - 1) - \{\Sigma_{j \neq i} (2p_j - t_j) \Sigma_{k \neq i,j} p_k\}/(N - 1)(N - 2)$.

THEOREM 4.4 (Hurwicz 1979a). In classical private goods environments such that all consumers' preferences are strictly increasing in good 1 and with at least three agents, $N \geq 3$, the set of Nash-equilibria allocations is equal to the set of Walrasian-equilibrium allocations.

PROOF. See Hurwicz (1979a).

Several remarks about the above two mechanisms are in order. First, each requires at least three traders. Hurwicz (1976), however, did define an efficient Nash mechanism for environments with only two traders. The outcome rule for that mechanism, however, is not balanced (i.e., the outcome function does not satisfy the condition $\Sigma h_i(m) \leq 0$), nor is it individually rational (see also Reichelstein 1984a). Second, the dimension of the message space used in the above mechanisms is $2N(L - 1)$. It is known that the minimal dimension needed to obtain Walrasian allocations under prescribed behavior is $N(L - 1)$ (see, for example, Mount and Reiter 1974). Thus, an open question of interest is whether it possible to design a Nash-efficient mechanism with the dimension of M being $N(L - 1)$.

Finally, neither of the above mechanisms is feasible in all environments, e, at all messages, m, in the sense that $h(m)$ may not be a feasible allocation for some message, environment pair (m,e). But, as we will see shortly, no mechanism exists that is balanced, nonparametric, feasible, and nontrivial (in other words, that has a nonzero outcome for some m).

Public Goods Environments. There are at least three specific mechanisms that are designed to allocate public goods in classical environments. The first, by Groves and Ledyard (1977), is defined as follows. The message space is $M_i = R^L$ for all i and $M = \Pi_i M_i$. The allocation rule is $h = (h_1(m), \ldots, h_N(m), y(m))$, where $y(m)$ is the chosen level of public goods and $h_i(m)$ is the amount of the numeraire good to be transferred to i.

$$y(m) = \sum_i m_i$$

$$h_i(m) = \frac{1}{N} y(m) + \frac{\gamma}{2} \left\{ \frac{N}{N-1} (m_i - \mu_{\sim i})^2 - \sigma(m_{\sim i}) \right\}$$

where $\gamma > 0$ is an arbitrary constant, $\mu_{\sim i} \equiv 1/(N-1)\ \Sigma_{j\neq i}m_j$, and $\sigma(m_{\sim i}) = 1/(N-2)\ \Sigma_{j\neq i}(m_j - \mu_{\sim i})^2$.

THEOREM 4.5 (Groves and Leyard 1977). In classical public goods environments with at least three agents, the mechanism defined above is an efficient Nash mechanism.

PROOF. (1) $\Sigma h_i(m) = y(m)$ for all m and thus the mechanism is balanced. (2) At Nash equilibria, $(u_{iy}/u_{ix}) - \{(1/N) + (N/(N-1))(m_i - \mu_{\sim i})\} = 0$. Summing over all i implies $\Sigma(u_{iy}/u_{ix}) - 1 = 0$, the Samuleson-Lindahl necessary conditions for efficiency. QED

In an interesting article, Bergstrom, Simon, and Titus (1983) show that this mechanism will have a large number of Nash-equilibrium messages, on the order of 2^{N-1}. Each will yield an efficient allocation but, as they point out, multiple equilibria may create problems for our justification of Nash behavior. In particular, Bergstrom et al. (1983, 167) state that "if there are multiple equilibria with differing distributions of utility, then individuals may have an incentive to falsify their preferences in order to drive the adjustment process to a preferred equilibrium." As we discuss below, in Section 4.1.3, there is no commonly accepted model of self-interested individual behavior of an agent confronted with an adjustment process. Until there is such a model, the implementability of this mechanism remains an open question.

Another property of this mechanism is that Nash-equilibrium allocations may leave a consumer worse off than at the initial endowment; that is, there may be consumers who would be better off not participating. In a mechanism by Hurwicz (1979a) this is avoided. His mechanism has the message space $M_i = \{(y_i,p_i) \in R^M \times R^M_{++}\}$ and $M = \Pi_i M_i$. The allocation rule is

$$h(m) = (h_1(m),\ldots,h_N(m),y(m)) \text{ where}$$

$$y(m) = (1/N)\sum y_i \text{ and}$$

$$h_i(m) = -|(1/N) + p_{i+1} - p_{i+2})|y(m) - p_i(y_i - y_{i+1})^2 - p_{i+1}(y_{i+1} - y_{i+2})^2$$

where $N + 1 = 1$ and $N + 2 = 2$.

THEOREM 4.6 (Hurwicz 1979a). In classical public goods environments with utility functions that are strictly increasing in the numeraire good and with at least three agents, the set of Nash-equilibrium allocations of the above mechanism are equivalent to the Lindahl allocations. Therefore it is an efficient Nash mechanism with individually rational allocations.

PROOF. (1) At Nash equilibria $p_i(y_i - y_{i+1}) = 0$ for all i. Therefore, (2) $h_i(m) = r_i(m)y(m)$ where $r_i(m) = (1/N) + p_{i+1} - p_{i+2}$. Also, (3) $(u_{iy}/u_{ix}) - r_i - 2p_i(y_i - y_{i+1}) = 0$ for all i. And, from (1), (4) $(u_{iy}/u_{ix}) = r_i$.. QED

An unfortunate property of this mechanism is its large message space. M is a $2NM$ dimensional space whereas the quadratic mechanism of Groves-Ledyard uses only an NM dimensional space for M. However, Walker (1981) discovered another mechanism which selects Lindahl allocations and which uses a smaller space for messages than does the mechanism of Hurwicz. Let $M_i = R^M$ and let $M = \Pi M_i$. Let $y(m) = \Sigma m_i$ and let $h_i(m) = [(1/N) + m_{i+1} - m_{i-1}]y(m)$ for each i.

THEOREM 4.7 (Walker 1981). In classical public goods environments with utility functions that are monotonic in the numeraire good and with at least three agents, the set of Nash-equilibrium allocations is equivalent to the set of Lindahl allocations.

PROOF. (1) $y(m) = \Sigma h_i(m)$, or balancedness, for all m. (2) At a Nash equilibrium, $h_i(m) = r_i(m)y(m)$ where $r_i(m) = (1/N) + m_{i+1} - m_{i-1}$. (3) At a Nash equilibrium, $(u_{iy}/u_{ix}) - r_i(m) = 0$. QED

To this point we know that, for classical environments, there exist continuous, balanced, nonparametric, individually rational, Nash-efficient mechanisms. There are also other mechanisms satisfying some, but not all, of these conditions. Unfortunately, although the equilibrium allocations are individually feasible, none of the above specific mechanisms are necessarily individually feasible at nonequilibrium messages. We say *unfortunately* for the same reason that we desired continuous outcome rules—in case of small errors in communication, implementation may require that nonequilibrium messages be used to compute the allocation. If this happens, it is very desirable to know that whatever allocation is chosen, it will be feasible for all agents. The next theorem sharpens some of the limits of mechanism design.

THEOREM 4.8 (Hurwicz, Maskin, and Postlewaite 1982). If a nonparametric outcome function is an efficient Nash mechanism and is individually feasible, then the message space for i must depend on w_i, i's initial endowment.

PROOF. Suppose h is an efficient, individually feasible Nash mechanism. Suppose for e it allocates $h(b^N(e)) \neq 0$. This will be true if w is not Pareto-efficient. Let $\tilde{m} = b^N(e;h)$. There is an i and a k such that $h_{ik}(\tilde{m}) = a < 0$. Consider the environment e' which is derived from e by lowering w_{ik} to c where $0 < c < -a$. Then h is not individually feasible in e'. QED

Allowing M_i to depend on w_i is formally equivalent to parameterizing the outcome function by w_i. Therefore, nonparametric mechanisms, i.e., those with nonparametric outcome functions and nonparametric message spaces, cannot be both individually feasible and Nash-efficient. This result is actually deeper: nontrivial, nonparametric mechanisms cannot be individually feasible.

To summarize the results in this section, we first recap some terminology. A mechanism is continuous if the outcome function $h: E \rightarrow A$ is continuous in an appropriate topology. A mechanism is balanced when allocating private goods if $\Sigma h_i(m) = 0$ for all m and balanced when allocating public goods if $T[\Sigma_i h_{ip}(m), h_y(m)] = 0$ where $h_{ip}(m)$ is the net addition to (or reduction in) i's endowment of private goods in the allocation $h(m)$ and $h_y(m)$ is the public goods allocation. A mechanism is individually feasible if $h_{ip}(m) \geq -w_i$ for all i and m and e. A mechanism is nonparametric if it is independent of e. A mechanism is Nash-efficient on E if $h(b^N(e;h))$ are Pareto-efficient allocations for all $e \in E$.

We have learned the following theorem.

THEOREM 4.9. In classical environments with at least two agents,

(a) there exist continuous, balanced, nonparametric, efficient Nash mechanisms,

(b) there do not exist (even with two agents) individually feasible, efficient Nash mechanisms.

All five specific mechanisms displayed above have desirable as well as undesirable properties. We touched on message size but did not discuss complexity, stability, and coalitional manipulability, to name just a few issues. Before effort is spent on further analysis of these five mechanisms, we would like to know how many others there are. That is, we would like to characterize the class of all efficient Nash mechanisms on classical environments. Although there have been several interesting papers written in this area, the characterizations remain incomplete.

In an interesting exposition of the Groves-Ledyard quadratic mechanism, Brock (1980) presents a method of generating an enormous class of efficient Nash mechanisms for public goods environments. His systematic approach also highlights what is needed to design such mechanisms. In particular, suppose the message spaces M_i and functions $y = y(m)$ and $T_i(m) = T_i$ for all i must satisfy (as is required for efficiency) balancedness (i.e., (1) $\Sigma_i T_i(m) = qy(m)$ for all m) and be such that Nash-equilibrium allocations are efficient; that is, if $u_{iy}(y,w_i - T_i)(dy/dm_i) - u_{ix_i}(y,w_i - T_i)(dT_i/dm_i) = 0$ for all i, then the Samuelson-Lindahl condition, $\Sigma_i[u_{iy}/u_{ix_i}] = q$, must hold. It is easy to see that this condition is satisfied if and only if (2) $\Sigma_i(dT_i/dm_i)/(dy/dm_i)] = q$ for all m. Now, as Brock (1980) showed, equations (1) and (2) can be used to generate innumerable efficient, Nash mechanisms. For example, let $M_i = R^K$ and let $y(m) = \Sigma_i m_i$. Then the functions, $T_i(m)$, must satisfy $\Sigma_i T_i(m) = q\Sigma_i m_i$ and $\Sigma_i dT_i/dm_i = q$ for all m and i. Suppose we try a series of polynomials for T_i. First consider $T_i = a_i + b_i m$. It is required that $\Sigma a_i + \Sigma b_i m = q\Sigma m_i$. Therefore, $\Sigma a_i = 0$ and $\Sigma_{ij} b_{ij} m_j = q\Sigma_j m_j$, for all m. The latter is possible if and only if $b_{ij} = b_j$ for all i and $\Sigma b_j = q$. If we were to require symmetry in the mechanism, then $a_i = 0$ for all i and $b_i = (1/N)q$ for each i. Therefore, taxes for all agents are equal, that is, $T_i(m) = (1/N)q\Sigma m_j$. It is easy to see that this mechanism satisfies the requirements: its Nash-equilibria allocations are Pareto-efficient. The difficulty with this particularly simple mechanism of equal taxation is that Nash equilibria

under the mechanism rarely exist. As in Hammond (1979), equilibria for this mechanism will exist if and only if there are fair Lindahl equilibria, that is, Lindahl equilibria such that all i have the same marginal rate of substitution. In most classical environments, such Lindahl equilibria simply do not exist. (Note: If the message spaces M_i are compact, then Nash equilibria may exist but will almost always be boundary points; that is, most m_i will be at the lower bound and only one agent i will effectively determine the allocation which will then not be Pareto-efficient since the Samuelson-Lindahl condition will not hold.)

Because of the existence problem with linear functions T_i, let us consider quadratic ones instead; that is, suppose $T_i = a_i + b_i + m'C^i m$ for all m and i. Now, for symmetric functions T_i it must be true that $\Sigma a_i = 0$, $b_{ij} = b_j$, $b_j = (1/N)q$ and $m'(\Sigma C^i)m = 0$ for all m to ensure balancedness. To ensure the Samuelson-Lindahl condition, we also need $\Sigma b_i + 2(\Sigma_i C^i_i)m = q$ and, therefore, $(\Sigma_i C^j_i)m = 0$.

It is straightforward, if somewhat tedious, to show that the quadratic rules of Groves-Ledyard satisfy these restrictions. We can obviously proceed in a similar way to cubics and higher-order polynomials. Since polynomials approximate most functions, we should be able to characterize all efficient Nash mechanisms this way; however, this remains to be done.

The approach of Brock (1980) can also be used to construct mechanisms that generate Lindahl allocations at their Nash equilibria. If the joint message m is a Nash equlibrium that produces a Lindahl allocation under a given mechanism, then the tax share for each agent i, $T_i(m)$, must equal $q_i(m_{\sim i})y(m)$ for Lindahl prices $q_i(m_{\sim i})$ that may depend on the messsages m_j of other agents, but not on agent i's own message. The Lindahl prices $q_i(m_{\sim i})$ by definition also sum to q. Thus, in place of the balancedness (1) and Samuelson-Lindahl conditions (2) above, we have the conditions: (3) $T_i(m) = q_i(m_{\sim i})y(m)$ and (4) $\Sigma q_i(m_{\sim i}) = q$. It is easy to see that (3) and (4) imply (1) and (2).

Suppose, then, that polynomial functions of $m_{\sim i}$ are constructed for $q_i(m_{\sim i})$. In the simplest case, that of constant functions, $q_i = a_i$ and the mechanism would pick at a Nash equilibrium those Lindahl allocations for which the marginal rates of substitution $u_{iy}/u_{ix_i} = a_i$. However, for any given environment e, for a prespecified set of a_i's, such Lindahl allocations would not likely exist.

Turning to the linear functions, $q_i(m_{\sim i}) = a_i + b_i m$ with $b_{ii} = 0$, for the symmetric case we need $a_i = (1/N)q$ and $\Sigma b_i = 0$. There are many such b_i; Walker (1981) found one particularly simple structure, where $b_{ij} = 1$ if $j = i + 1$, $b_{ij} = -1$ if $j = i - 2$, and $b_{ij} = 0$ otherwise. It is interesting to note that the form of q_i is independent of the form of $y(m)$. As long as $dy/dm_i = 0$ for all m and i, any function $y(m)$ will do. We also know from Hurwicz

(1979b) that the form $T_i = q_i(m_{\sim i})y(m)$ is not necessary if h is to generate Lindahl allocations. His mechanism, as defined above, has the form $T_i(m) = q_i(p_{\sim i})y(r) + R_i(r,p)$, where $m = (r,p)$. We note, however, that in equilibrium $R_i(r,p) = 0$, leaving the form $T_i = q_i(m_{\sim i})y(m)$. It remains an open question whether all mechanisms whose Nash allocations are Lindahl have essentially only this structure.

Turning now to private goods environments and proceeding as above, we immediately run into a problem. As with Brock's (1980) approach for public goods mechanisms, we look for functions $x_i(m)$ and $T_i(m)$ for all i such that $\Sigma x_i(m) = 0$, $\Sigma T_i(m) = 0$, and the Nash-equilibrium allocation, $(w_i - T_i, x_i)_i$, is Pareto-efficient. These functions must then satisfy $\Sigma x_i(m) = 0$ and $\Sigma T_i(m) = 0$ (the balancedness condition), and if $(du/dx_i)(dx_i/dm_i) - (du/dy)(dT_i/dm_i) = 0$ for all i, then there is a P such that $(du/dx_i)(du/dy_i) = P$ for all i. Given balancedness, we therefore require that $(dT_i(m)/dm_i)/(dx_i(m)/dm_i) = P(m)$ for all i and all m. Looking first at $x_i(m)$, suppose that $M_i = R^{L-1}$ and $x_i(m) = m_i$, a proposed trade. This is not balanced since, in general, $\Sigma m_i \neq 0$. To balance this, we can subtract the average surplus, $\Sigma m_i/N$, and get $x_i(m) = m_i - [\Sigma m_i/N]$. We can rewrite this as $x_i(m) = ((N-1)/N)(m_i - r_{\sim i})$, where $r_{\sim i} = \Sigma_{h \neq i} m_h/(N-1)$. Rescaling m gives $x_i(m) = m_i - r_{\sim i}$, which is Hurwicz's rule. Now given this rule for $x_i(m)$, we can take the approach of Brock (1980) to characterize T_i. Suppose T_i is a polynomial, $T_i = a_i + b_i m + \ldots$. Then we require that $\Sigma a_i = 0$, $\Sigma b_i = 0$, $b_{ii} = P(m)$ for all m. Thus $b_{ii} = b^*$ for all i. If we also require symmetry, then, in addition, $a_i = 0$ for all i, and $b_{ij} = -1/(N-1)b^*$ for all $i \neq j$. Thus, $T_i(m) = b^*(m_i - r_{\sim i}) = b^* x_i(m)$. The problem with these rules is now obvious: we need to know the Walrasian price to know b^* if we wish an equilibrium to exist, but the designer does not have that information. In fact, it is impossible to find any $T_i(m)$ that do the job we want. The message space M is simply not big enough. As we indicated above, if Nash allocations are to be Walrasian and h is to be differentiable, then M must have dimension at least $N(L-1)$. In the public goods model we assumed that the vector of public goods prices was known to the designer and that there was only one private good that also had a known price equal to unity. That still left an incentive problem in those models. If relative prices were known in a private goods environment, however, there would then be no incentive problem. Let $x_i(m) = m_i$, $T_i(m) = -pm_i$, and $h_i(m) = (x_i(m), T_i(m))$. If p is indeed a Walrasian relative price for e, then this is an efficient Nash mechanism. The mechanism is, of course, parametric since the relative price p will depend on the environment e, and assuming that it is known is assuming away the incentive problem entirely. Presumably one can also design mechanisms, for public goods environments, that not only choose the level of public goods but also choose prices and the level of private goods; however, this remains to be done.

In an important paper Hurwicz (1979a) took an entirely different approach to the characterization of efficient Nash mechanisms. He was able to demonstrate that, under fairly reasonable conditions, if one wants to design a mechanism whose Nash-equilibrium allocations were Pareto-efficient and individually rational on the classical private goods environments, then those Nash-equilibrium allocations must coincide with the Walrasian allocations. A similar result is obtained for public goods environments. That is, remarkably, any mechanism designed to produce individually rational, efficient, Nash equlibria would have to yield Walrasian or Lindahl allocations. Attempts to obtain other allocations would be fruitless. Similar results can be found in Schmeidler (1982a).

The precise nature of this amazing result is as follows:

THEOREM 4.10 (Hurwicz 1979a). Given an allocation mechanism h, suppose that the Nash equilibria-allocations, $h(b^N(e;h))$, are contained in the set of individually rational, Pareto-efficient allocations for e for each e in E.

(A) If $b^N(e;h)$ is nonempty and upper semicontinuous on E, then, for each $e \in E$, the Walrasian allocations of e are contained in $h(b^N(e;h))$, the Nash-equilibrium allocations.

(B) Define $B_i(m) = h_i(m/M_i) - R^L_+$. (This is the set of consumptions i can unilaterally get to from the message m.) If $B_i(m)$ is convex for all i and all $m \in M$, then for all $e \in E$ the allocations in $h(b^N(e;h))$ that are interior are also Walrasian. (An allocation is interior if, for all i, $x_i + w_i \gg 0$.)

PROOF. See Hurwicz (1979).

Continuity of the Nash-equilibrium correspondence is a very desirable property for an allocation mechanism for the same reason that continuity of h is desirable. Small errors in observation or calculation will, then, not lead to large perturbations in allocations. Convexity of the sets $B_i(m)$ is desirable because it is sufficient to ensure that the best response functions of the i are upper semicontinuous, a property used to get upper semicontinuity of the Nash correspondence. Thus both properties required by Hurwicz are reasonable.[11] They are satisfied by all five examples we have presented in this

[11] In Reichelstein (1984b) it is shown that without the convexity requirement, there are many more implementable choice rules; in particular, the one that selects individually rational Pareto-optima is fully implementable (Corollary to Proposition 3.1).

section. Of these mechanisms, only the quadratic rules of Groves-Ledyard do not have Walrasian or Lindahl allocations as Nash outcomes. The reason is that the mechanism does not satisfy the requirement of individual rationality.

To summarize: individually rational, efficient Nash mechanisms with continuity and minimal message spaces are those that produce Walrasian or Lindahl allocations. In private goods environments the appropriate mechanisms are those of the following general form: Let $M_i = R^{L-1} \times R^{k_i}$ where $k_i \geq L - 1$. Let $m_i = (s_i, p_i)$, and define $x_i(m) = s_i - v_{\sim i}$ where

$$v_{\sim i} = \sum_{j \neq 1} \frac{1}{N-1} s_j \text{ and } t_i(m) = f_i(m_{\sim i})x_i(m) + T_i(s_{\sim i}, p),$$

$$\text{where } \min_p T_i(s_{\sim i}, p) = 0 \text{ for all } s_{\sim i}.$$

In public goods environments, assuming relative output prices are known, the appropriate mechanisms are those of the following general form: Let $M_i = R^K$ and define $y(m) = \Sigma m_i$ and $T_i(m) = f_i(m_{\sim i})y(m)$ where $\Sigma f_i(m_{\sim i}) = q$. If relative prices also need to be determined by the mechanism, then a larger message space will be needed.

If we forego individual rationality, other mechanisms, such as the quadratic rules for public goods, become available. And, if we are willing to forego continuity, we may be able to reduce the size of the message space; full characterizations, though, remain to be done.[12]

4.1.3. Manipulative Equilibria

The assumption that behavior is modeled by Bayes equilibrium or Nash equilibrium is, by far, the most common in the literature. However, it has been argued by some that the assumption of Nash behavior views participants to be somewhat naive. Thus it might be expected that more clever agents would find out that they could improve on their allocation with more sophisticated play. Since we always assume that players in these games are at least as clever as the modelers, this observation raises some serious issues for mechanism design which must be considered.

To illustrate this problem, consider a two-person environment with the mechanism h, which is known to be an efficient Nash mechanism. That is, $h(b^N(e;h))$ is Pareto-efficient in e for all $e \in E$. Now consider the indirect utility function of m given by $v_i(m) = u(h(m),e_i)$ for each i. We can plot

[12] In a related paper, Aghion (1984) has shown that the set of continuous mechanisms whose equilibria are inefficient is dense in the set of continuous mechanisms. This, however, still leaves more than enough candidates whose equilibria are efficient.

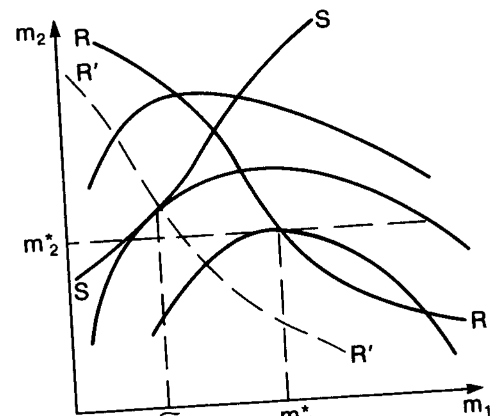

Figure 2.6. Example showing that a manipulative Nash equilibrium cannot be Pareto optimal.

the indifference curves for $i = 1$ as in Figure 2.6. Although it may not be the fully rational game-theoretic equilibrium, one way to think of a single agent's behavior in the infinite iteration process is as a myopic maximizer at each iteration. If we let $m_2 = m_2^*$, then we can find the m_1 that maximizes v_1 given m_2^*. This is m_1^* in the diagram. In fact, we can plot all such best replays as the locus RR in Figure 2.6. Similarly, we can plot the best responses of agent 2 as the locus SS. The intersection of the SS and RR curves gives us the Nash-equilibrium messages, the only stationary points consistent with this myopic behavior. But agent 1 has available a better strategy if 1 can identify the function SS. If 2 will indeed respond as predicted by the Nash assumption, then if 1 chooses $\bar{m}_1$ to maximize $v(m)$ subject to the joint strategy (m_1, m_2) lying on SS, 1 will be better off. Of course, it is possible for 1 to disguise this manipulative behavior. Instead of choosing a single message, 1 can pretend to have preferences that yield the pseudoresponse curve $R'R'$. That is, 1 chooses e_1' to maximize his indirect utility $u(h(b^N(e/e_1'));e_1)$. Then, if 2 behaves myopically according to SS and 1 behaves myopically according to the response function for e_1', the same result will occur as if 1 were to choose $\bar{m}_1$.[13]

If all agents attempt this level of sophisticated behavior in our general

[13] Economists will note that these are old concepts in the literature. The Nash equilibrium was proposed by Cournot in modeling oligopoly. The sophisticated resonse was proposed by Stackleberg to model leader-follower price-setting behavior.

model of resource allocation, an appropriate equilibrium concept for this situation would be what Hurwicz (1975) called the *manipulative nash equilibrium*. Formally, m^* is a manipulative nash equilibrium (MNE) if $m^* = b^N(\bar{e},h)$ where $u[h(b^N(\bar{e},h));e_i] \geq u[h(b^N(\bar{e}/e_i',h));e_i]$ for all $e_i' \in E_i$ and $b^N(\cdot,h)$ is the (naive) Nash-behavior rule given the mechanism h. *Manipulative Nash Behavior* $b^M(e;h)$ then is defined as the mapping from environments e to the *MNE* for e. This concept may be interpreted as the equilibrium joint message that would result if all agents behave during the iterations as if they are (naive) Nash players but strategically choose the personal characteristics $\bar{e}_i$ that generates the best Nash response for them, given that the others are following Nash behavior as well. Alternatively, we could imagine that the given mechanism h is implemented as a direct revelation mechanism h' in which each agent is asked for the characteristic $\bar{e}_i$ and the allocation then calculated is that which would be given by the original mechanism h if the joint message $\hat{m}$ is the (naive) Nash equilibrium for the stated characteristics $\bar{e}$.

The implications of manipulative Nash behavior for efficient mechanism design are negative as established by Hurwicz in the following theorem:

THEOREM 4.11 (Hurwicz 1981). There are no mechanisms h such that the manipulative Nash allocations, $h[b^M(e;h)]$, are Pareto-efficient on the classical environments.

PROOF. See Hurwicz (1981).

A corollary to this theorem is that even if a mechanism is an efficient Nash mechanism, its manipulative Nash allocations are not Pareto-efficient. Thus, even if a mechanism is designed to effectively channel the incentives under (naive or nonmanipulative) Nash behavior, if agents are sophisticated and adopt manipulative Nash behavior, the effect will be unsuccessful (See Thomson (1984) for some results concerning likely outcomes under manipulative Nash behavior in environments with quasi-linear preferences.)

Although it may appear that no mechanism can prevent sophisticated manipulation from leading to inefficiency, we should note that the definition of manipulative behavior above is based on the assumption that the underlying, naive behavior is Nash. We can generalize the above notion of manipulative behavior by considering other naive models. For example, let $b(e;h)$ be an arbitrary model of behavior for the mechanism h. If this model is correct, then the outcome will be $h(b(e;h))$. But clearly, we can also use this model to compute an optimal manipulation. We call $b^M(e;h,b)$ the manipulative-behavior model, given h and b, *if* $b^M: E \to A$ where $b^M(e;h,b) = h(b(e^*;h,b))$ and $u[h(b(e^*;h));e_i] \geq u[h(b(e^*/e_i';b));e_i]$ for all i and $e_i' \in E_i$. That is, e^* is a Nash equilibrium of the direct revelation mechanism $f(e';h,b) = h(b(e',h))$. This generalizes our previous definition in the sense that $b = b^N$, or Nash behavior, is only one possible b. To see that the assumed behavior b is important, consider the following theorem which stands in direct contrast to Hurwicz's result.

THEOREM 4.12 (Reichelstein 1982). If h is an efficient Nash mechanism and if the postulated behavior is $b(e;h) = [b_1(e_1;h),\ldots,b_N(e_N;h)]$ where $b_i(\cdot\,;h)$: $E_i \to M_i$ is an onto map, then the allocations $h(b^M(e;h,b))$ are Pareto-efficient.

PROOF. See Reichelstein (1982).

(Note that the postulated naive behavior cannot be Nash behavior!) This theorem asserts that if each agent postulates that all others' naive behavior depends only on their own characteristic e_i, then the manipulative-equilibrium allocation will be efficient if h is a Nash-efficient mechanism. Notice that if all i follow the postulated naive behavior b and do not manipulate, then $h(b(e;h))$ cannot be efficient over all E. The main problem with this result is that it does not make sense to us to assume that sophisticated behavior is Nash when naive behavior is not. The players' postulate about each other's naive behavior should be consistent with the assumed level of sophistication.

It is important to note that with arbitrary outcome function-behavior rule pairs (h,b), the analysis becomes simply that of direct revelation mechanisms. That is, given (h,b), define $f(e) = h(b(e))$. Any question about the manipulative performance of h, given the assumption that the naive behavior is b, is equivalent to the same question about the Nash-equilibrium behavior of the mechanism with $M = E$ and $h'(m) = f(m)$. For example, (h,b) has efficient manipulative equilibria if and only if f is an efficient (direct revelation) Nash mechanism.

An interesting result that follows from the above is:

THEOREM 4.13. There exist individually rational direct revelation mechanisms for classical environments such that their Nash equilibria are Pareto-efficient. However, at the Nash equilibria, the agents' (equilibrium) messages $e_i^* = m_i^*$ are not, in general, coincident with their true characteristics e_i.

PROOF. Follows from Theorem 4.12 (Reichelstein), Theorem 3.1 (Hurwicz), and Theorem 3.2 (Ledyard and Roberts).

In view of these results, it is important to ask how likely manipulative behavior is. For one-shot implementations of mechanisms it is hard to find any rationalizations for manipulative behavior (as defined here); its plausibility surely depends on some form of iterative implementation of a mechanism. However, when a mechanism is iteratively implemented, many aspects that are not included in a normal form description of the mechanism become important. As Smith (1982) has emphasized for experimental economics, detailed instructions must be specified to implement any specific mechanism. Suppose, for example, that we wished to use the Walker mechanism to make a public goods choice. The instructions would have to specify:

(1) *The language of communication*. If there is a single public good, then the message any agent sends to the experimenter (or the *center* or the *auctioneer*) is a single real number, say m_i. Then, the experimenter will return a message to each agent. In this particular mechanism, this message may be all others' responses, or just the responses (m_{i-1}, m_{i+1}), or a number, say q_i, where $q_i = (1/N)q + m_{i-1} - m_{i+1}$. Each of these responses by the center represents, in principle, a different institution.
(2) *The timing and addresses of communication*. All agents must know when to communicate their responses and when to expect to receive responses. The agents must also know to whom to send a message and from whom to receive one. In the above centralized institution, communication proceeds in orderly iterations where each agent sends a message m_i to the experimenter who, after receiving all responses, sends a (possibly different) message simultaneously to each agent. It is this step that begins to identify what each agent knows, other than e_i, at each step of the process.
(3) *A stopping rule*. The communication process must stop sometime, and the nature and timing of its cessation must be specified. One possible example, in this case, is to state that the iterative process described in (2) will end either (a) when all agents match their previous messages two times in a row or (b) when, say, thirty iterations have elapsed, whichever occurs first. It should be obvious that if (b) is deleted, we would have a radically different process: different stopping rules may lead to dramatically different outcomes.
(4) *An outcome rule*. Each agent should know what action is to be implemented after the communication ceases. One rule for the Walker mechanism could be as follows: If (3a) is the reason communication ended, then the experimenter takes the last response of each agent and computes as follows: the public goods level chosen is $y = \Sigma m_i$, and each i pays $T_i = ((1/N)q + m_{i-1} - m_{i+1})y$. This is simply the rule $(x_1, \ldots, x_N, y) = h(m)$ If (3b) is the reason communication ended, then let $(x_1, \ldots, x_N, y) = 0$. That is, if there is no agreement, the *status quo* is the implemented allocation.

If the experimenter or the mechanism designer did not specify all of the above components of the process, the mechanism h could not produce a choice. Thus the entire process is necessary. It is also important since it is highly unlikely that, in practice, different versions of (2) and (3) will produce the same allocation even if (1) and (4) are identical versions of the same mechanism. This means that the behavior, $b(e;h)$, may be different depending on the (generally unspecified) components of the process. The normal form of the mechanism may be an insufficient description for a thorough analysis of the performance of designed mechanisms. Thus, a deeper analysis of manipulative behavior must depend on a rigorous analysis of behavior when details of the extensive form are specified.[14] In particular, given the process (1) to (4) above, each agent is faced with a complex, sequential, incomplete information game. Manipulation can be achieved only through the sequence of messages one sends, and the real issue is whether the outcome attained is near to the normal-form Nash equilibrium.[15] This is both an empirical and a theoretical issue: empirical in the sense that what we want to know is how agents will actually behave when confronted with a new mechanism, and theoretical in the sense that we want to know how agents should behave when confronted with a new mechanism. It is our view that when this type of analysis is done, it will be discovered that the specifics of (1) to (4) will be very important, and that there are processes in which sophisticated manipulation is virtually impossible because of the informational requirements of such a strategy. Of course, these remain open questions.

4.1.4. Other Possibilities

The design and evaluation of efficient mechanisms have been carried out for other types of presumed behavior. We include two of the more common types in this section for completeness.

The main results in the area of maximin behavior are attributable to Thomson (1979). Maximin behavior results from an agent's hypothesis that, for each message chosen, the other players will jointly choose their messages to minimize his payoff, given his message. Under this hypothesis, the agent

[14]It should be noted that in his initial papers in this area, Hurwicz carefully specified the iterative process of communication. Many of those models of resource-allocation mechanisms were complete processes in the sense that (1), (2), and (4) were explicitly specified. The stopping rules (3) were implicit, but, since only "equilibrium" messages were considered, they were probably something like (a) above. Actually, the initial specifications of Hurwicz went further as each agent was also told which response to make given the message of the center and his own characteristic. It was the realization that the designer could not guarantee that the specified rules would be followed, since the designer did not know the particular e_i of each agent, that led to the formulation of the incentive problem as we have presented it in this chapter.

[15]Indeed the fact that the Complete Information Nash equilibria are not in general Manipulative Nash equilibria leads us to suspect that detailed analysis of the incomplete information, repeated game will show that the outcome will not be near to the outcome attained at the normal-form Nash-equilibrium messages.

then chooses a message that maximizes this minimum payoff. Thomson has shown that for the subset of environments with quasi-linear preferences, there exists an efficient maximin mechanism. Results for other environments are unknown. The maximin behavioral rule arises naturally in the context of two-person zero-sum games but does not seem to be compelling in the N-person nonzero-sum games that we consider in this chapter. Only if the individuals are infinitely risk averse or extremely paranoid should they be expected to behave as required by the maximin hypothesis.

The other behavioral rule rests on an assumption of myopia that arises naturally in the context of planning procedures. The research in this area for private goods environments dates to the debate over the relative merits of socialist planning and free markets. Refer to the planning models of Mal-invaud (1967), Weitzman (1970), and Heal (1973), and to the survey of Hurwicz (1973) for a summary of this literature. For public goods environments, the initial literature consists of papers by Malinvaud (1971) and by Dreze and de la Vallée-Poussin (1971). Robert's work in this volume (Chapter 14) provides an excellent summary of the extensive literature that followed from these original papers. Our remarks here are intended merely to provide a bridge between our chapter and his. The general structure of these planning procedures bears a close resemblance to the processes described in the previous section on manipulation. The main formal difference is that, in general, the outcome rule of a planning procedure depends on the entire sequence of messages sent, not just on the last message.

Formally, a (discrete time) planning procedure is a language, M_i, for each i, a state space, S, and an explicit iterative process of communication. This iterative process is modeled as

$$s(t+1) = f(s(t), m(t)), \text{ for dates } t = 1, 2, \ldots \quad (1)$$

The final allocation determined by the process is given by an outcome rule, $h(s) = a$, and a stopping rule which defines that state $s(t)$ to which h is to be applied. If $f(\cdot)$ is defined so that $s(t+1) = m(t)$, then this is an allocation mechanism with h as the outcome function. The generalized form (1) allows planning procedures that are indirect control devices; that is, instead of directly determining the outcome through h, the agents directly control s and indirectly control the outcome through the cumulative effect of m on s. This allows for smoother but possibly less-rapid convergence to the desired allocation.

In the Dreze–de la Vallée-Poussin and Malinvaud (DVM) mechanisms, m_i is a vector of marginal rates of substitution, or marginal willingnesses to pay, and s is the "proposed" allocation. (Although in the original papers, (1) is in continuous form, that is, $ds/dt = f(s,m)$, we consider here the

discrete-time versions.) In these models the appropriate response rule of each i is specified as

$$m_i(t) = g(s(t), e_i), \quad (2)$$

the assumption being that each agent will follow the rules. Of course, it was realized that agents might not, and the incentive properties of the procedures were analyzed in those papers under different behavioral assumptions. The basic approach is to assume that, at each iteration, an agent is only concerned with the current increase in utility. This is similar to assuming that the agent thinks the current iteration is the last and that $h(s(t+1))$ will be implemented. We can analyze this myopic, or local, behavior in the same way that we have analyzed the global model. We give two examples and refer the reader to Roberts (1985) for others.

In their 1971 paper Dreze and de la Vallée-Poussin presented a planning procedure with the property that if agents adopt local maximin behavior, then they will report their true marginal rates of substitution and the procedure will then converge to a Pareto-efficient allocation. That is, in classical public goods environments with a finite set of agents, there exists an efficient, local maximin procedure. Roberts (1979) has shown that if agents use local Nash behavior (that is, $m(t)$ is a Nash equilibrium for the local game), then the same DVM procedure will converge to Pareto-efficient allocations, although at a slower rate. That is, in classical public goods environments with a finite set of agents, there exists an efficient, local Nash procedure. One drawback of the local Nash assumption is that it is not clear how agents are to determine these Nash responses since the procedure does not explicitly allow for a sequence of iterations of messages before $m(t)$ is determined. Thus the justification we used for relying on the (global) Nash assumption is missing for the local Nash assumption. If agents look ahead instead of behaving myopically, we are in a similar situation to that described in the last section on manipulative equilibria of mechanisms.

The form of the planning procedure (1) provides the agents with a dynamic game where the strategies are $m_i(\cdot)$, functions of t from 0 to T, the transition equations are given by (1), and the payoffs are $u(h(s(T)), e_i)$ for each i. If i knew the strategies of the others, i could compute a best-response strategy by solving an optimal control problem, but as in the case of the manipulation of allocation mechanisms, each agent only knows his own characteristic and the sequence of reports from the center. This is thus a complicated, sequential, incomplete information game. Truchon (1984) has shown, for quasi-linear preferences, that there is a discrete time process whose perfect Nash equilibrium converges to an efficient allocation. Roberts (this volume) has also begun an analysis of this problem but with essentially negative results. He concludes that the informational savings involved in adopting an iterative procedure can be realized only at a cost of lost efficiency. In other words, there may be no planning procedures whose sequential equilibrium allocations are Pareto-efficient. See Roberts (this volume) for a discussion of this and possible future research.

4.2. *Parametric Mechanisms*

We have seen that if we restrict ourselves to nonparametric outcome functions, then it is impossible to design efficient, dominant-strategy mechanisms if the class of environments is reasonably large. It is, however, possible to design efficient Nash mechanisms, but, again, if the class of environments is reasonably large, then the outcome rules for these mechanisms cannot be fully feasible. The logical next step in the development of the theory of allocation mechanisms is to explore what can be accomplished if we allow parametric outcome functions. If information about the environment e, or about the class of environments E, can be used in the outcome function, even though that information was not transmitted as a message by any agent, then we say that function is parametric. We denote such functions by $h(m,I(e))$, where $I(e)$ denotes that information about e which is to be used by h. This information usually is used in one of two forms: either direct information about the specific environment e, or indirect information about the range of possible e in E in the form of a probability measure on E. Let us look at each of these.

4.2.1. Direct Information

One of the drawbacks of many of the mechanisms whose Nash-equilibrium allocations are Pareto-efficient is that, at non-Nash equilibrium messages, the outcome rule may compute an allocation that is not feasible for some i, even if that rule is continuous and balanced. (See Theorem 4.8 above.) If such a mechanism were actually used in, say, an iterative process that was terminated prior to the attainment of a full Nash equilibrium, then some agents might not be able to survive on the indicated allocation. Hurwicz, Maskin, and Postlewaite (1982) overcame this problem with the use of parametric outcome rules. They incorporated direct information about the initial endowments into the outcome function h by allowing the admissible strategies to depend on the initial endowment w. The function h maps strategies into actions as follows: h: $S(e) \rightarrow A$, where $S(e) = \Pi_i(S_i(e_i)$ and $S_i(e_i) = M_i \times [0,w_i]$. This form of parametric function possesses two desirable features. First, it retains much informational decentralization since no agent need know the characteristics of the others. Because of this feature, Hurwicz, Maskin, and Postlewaite called these outcome functions *decentralized parametric*. Second, the center need only be able to verify that each agent

has at least as much initial endowment as reported. Endowments are, in principle, capable of being audited; in this case, however, agents need only produce the claimed allocation. Ignoring the costs of that auditing procedure, we find this class of parametric functions to be a natural candidate for consideration in our search for efficient mechanisms.

In their paper Hurwicz, Maskin, and Postlewaite (1982) consider this class of mechanisms in detail and provide a list of possibilities. Although they consider both private and public goods environments, let us restrict our attention, for now, to classical private goods environments. The results for public goods environments are the same as those below if we replace the identifier "Walrasian" with "Lindahl." The first result follows from a clever example and a theorem of Maskin (1977) on necessary conditions for implementability when endowments are known by the designer.

THEOREM 4.14 (Hurwicz, Maskin, and Postlewaite 1982). There is no decentralized parametric outcome function such that $h(b^N(e;h)) = W(e)$ on the class of classical private goods environments.

PROOF. See Hurwicz, Maskin, and Postlewaite (1982).[16]

The difficulty in producing the desired mechanism arises in those environments in which the Walrasian allocation is on the boundary of the feasible set of allocations. Hurwicz, Maskin, and Postlewaite show, however, that it is possible to adjust for this anomaly and to produce a decentralized parametric mechanism that is feasible at all messages and that selects efficient Walrasian allocations when these are interior and efficient allocations otherwise. Before presenting their mechanism, consider the following useful performance correspondence. The Constrained Walrasian correspondence, $CW(e)$, is defined as follows: $CW(e) = \{(x_1,\ldots,x_N) \mid$ there is p such that $\Sigma_i CD_i(p) = 0$, $x_i \in CD_i(p)$ for all $i\}$, where $CD_i(p) = \{x_i \mid px_i = pw_i,\ u(x_i,e_i) \geq u(z,e_i)$ for all z such that $pz = pw_i$ and $z \leq \Sigma_h w_h\}$. Note that these are the market equilibrium allocations when individual demand choices are constrained so that agent i's demand does not require more of any commodity than is available to the entire economy. Letting $P(e)$ be the Pareto-efficient allocations for e and $IP(e)$ be those a in $P(e)$ such that $u(a,e_i) \geq u(w_i,e_i)$ for all i—that is, the individually rational allocations—we can describe the relationships between these various performance correspondences as follows:

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[16] They actually prove more: even if the designer knows the initial endowments, the Walrasian correspondence is not implementable since it is not monotonic. A performance correspondence, $P(e)$, is said to be *implementable* (in Nash equilibria on E) if and only if there is an outcome function h such that $h(b^N)e;h)) \subseteq P(e)$ for all e in E. A performance correspondence, $P(e)$, is *monotonic* if, given $a \in P(e)$, for any $e' \in E$ such that whenever $u(z,e_i) \leq u(a,e_i)$ implies $u(z,e_i') \leq u(a,e_i')$, then $a \in P(e')$.

$W(e) \subseteq CW(e) \subseteq IP(e) \subseteq P(e)$ for all $e \in E$. Note that $W(e) = CW(e)$ whenever $a \in W(e)$ implies that $a_i > 0$ for all i, or, less restrictively, whenever $a_i < w$ for all i. Given these concepts, we can state the following:

THEOREM 4.15 (Hurwicz, Maskin and Postlewaite 1982). There is a decentralized parametric allocation mechanism, with finite dimensional messages, for which $h(b^N(e;h)) = CW(e)$ on the classical private goods environments.

PROOF. See Appendix to Section 4 for a sketch of the proof.

The mechanism described in the proof of this theorem is such that its Nash-equilibrium allocations are constrained Walrasian allocations as long as endowment misrepresentations can only be less than the true endowment; that is, endowments can be withheld. Hurwicz, Maskin, and Postlewaite also provide a proof of the above theorem if withheld endowments must be destroyed. Such would be the case if, for example, endowments not only can be required to be shown but also can be found if they are withheld. In practice, the former is similar to the requirement that a buyer demonstrate a sufficient bank balance prior to purchase, whereas the latter is similar to an IRS tax audit. The former is clearly less expensive than the latter.

The Hurwicz, Maskin, and Postlewaite theorem implies the following simple corollary:

COROLLARY

(a) There exist decentralized parametric, feasible Nash-efficient allocation mechanisms on the class of classical private goods environments.

(b) There exist decentralized parametric, feasible Nash-efficient allocation mechanisms whose allocations are individually rational.

Also, a slight modification of Hurwicz's theorem characterizing Nash-efficient mechanisms establishes Theorem 4.16.

THEOREM 4.16 (Hurwicz 1979). If h is a continuous, decentralized parametric outcome function such that the Nash-equilibrium allocations are individually rational and Pareto-efficient, then the constrained Walrasian allocations are contained in the Nash-equilibrium allocations. That is, $CW(e)$ is the smallest individually rational, Pareto-efficient performance correspondence that is implementable with a decentralized mechanism.

PROOF. See Hurwicz (1979).

One drawback of the mechanism used to demonstrate existence of Nash efficiency above is that the function, $h(s,v)$, defined in Appendix 4 by equations (1.1) to (1.3), is not continuous in either s or v. There is, however, a mechanism established by Postlewaite (as reported by Schmeidler 1982b) that is continuous in s and which can be used to establish similar results.

THEOREM 4.17 (Postlewaite and Wettstein 1983). There exist decentralizable parametric, feasible Nash-efficient mechanisms, $h(s,v)$, that are continuous in s.

PROOF. See Appendix 4 for a sketch.

It remains an open question whether there exist continuous (in all m) decentralized parametric, feasible Nash-efficient allocation mechanisms for the classical private goods economies.

4.2.2. Indirect Information

In his seminal article of 1972, Hurwicz considered briefly a model of the design problem which is a little different than those we have considered here. In his model, there was a welfare function on outcomes, W, so that the welfare associated with the mechanism h, given the behavior b, in the environment e could be expressed as $w = W(h(b(e;h)),e)$. A variant of this model allowed resources to be expended in the operation of the mechanism. Thus, for this variant, if $r(h,e)$ is the cost of operating mechanism h in the environment e, $w = W(h(b(h;e)) - r(h,e),e) = w(h,b,e)$ The designer's problem was then recognized to be the statistical decision problem to maximize w over the set E by choosing h. If a probability measure over E were available to the designer, the designer might choose h to maximize expected welfare. This important observation by Hurwicz foreshadowed the literature on optimal auction design and provided the basis for the inclusion of what we call indirect information into the design of the allocation mechanism. This information will be in the form of a probability measure on E, which is common knowledge to all agents and to the designer. This is indirect information in that it is not directly auditable since, just as in the case of preferences, probability beliefs are nonobservable and may be only indirectly inferred from evidence about actions. This inability to audit beliefs raises serious questions about the efficacy of mechanisms that are designed, assuming knowledge of those beliefs. However, since the literature in this area is extensive and others do not share these doubts, we summarize the results under the assumption that the mechanism can indeed use indirect parametric information.

Suppose that $P(E)$ is the set of probability measures on E. Consider parametric mechanisms, $h(m_1,\ldots,m_N,p)$ where $p \in P(E)$. In the language of incomplete information games, we assume that p is common knowledge to the mechanism designer, the mechanism operator, and all the agents. At issue is the same question we have addressed all along: given some behavior that is consistent with the agents' incentives, are there any mechanisms of this type such that some specified performance results? A natural assumption of reduced form behavior in this type of mechanism, given the common knowledge assumption, is that of Bayes equilibrium. This was defined earlier in section 4.1.1 as follows: A strategy for i is a function d_i: $E_i \rightarrow M_i$. A Bayes equilibrium, given h and p, is a vector of strategies $(d_1, \ldots, d_N)$ such that for every i and every e_i, $d_i(e_i,p)$ solves Max $\int_{E_{\sim i}} u(h([m_1(e_1), \ldots, m_N(e_N)]/m_i,p),e_i)dp(e|e_i)$ where $E_{\sim i} \equiv \Pi_{j \neq i} E_j$. Given a Bayes equilibrium, d, the outcome is $a = h(d(e,p),p)$ for each e in E. Of course, multiple Bayes equilibria[17] can be a problem in the same way as multiple Nash equilibria, although for our purposes there is no difficulty. For example, if we want $h(d(e,p),p)$ to be efficient and there are multiple equilibria, let $D(e,p)$ be the set of equilibria. Then we simply ask if $h(d,p)$ is efficient for all $d \in D(e,p)$.

One question that immediately suggests itself is whether there is any indirect information mechanism that is also efficient at Bayes equilibria, in the sense that $h(d(e,p),p)$ is Pareto-efficient for all e in E. There are at best some partial answers. In classical public goods environments, if we further restrict preferences to be quasi-linear, there exists an efficient parametric mechanism for a subset of possible probability measures.

THEOREM 4.18 (D'Aspremont and Gerard-Varet 1979). If E is the set of classical public goods environments with quasi-linear preferences and p is a probability measure on E such that e_i is distributed independently of $e_{\sim i}$ for all i, then there exists a direct-revelation mechanism $h:(M,p) \rightarrow A$ such that $e = d(e,p)$ and $h(e,p)$ is Pareto-efficient for all e in E.

PROOF. See Appendix 4 for a sketch of the proof. This theorem was also discovered independently by Arrow (1979).

It remains an open question, as far as we are aware, whether there exist parametric mechanisms that are efficient at all Bayes equilibria for all classical environments, although Postlewaite and Schmeidler (1985) have made progress under some information structures.

4.3. *Summary*

Combining the results in the previous sections, we can summarize the state of knowledge concerning the possibilities for the design of efficient, incentive-sensitive mechanisms in finite environments.

THEOREM 4.19. In classical environments (both private and public goods) with at least two agents:

(a) There do not exist nonparametric, efficient Bayes mechanisms (Section 4.1.1).
(b) It is an open question whether there exist parametric, efficient Bayes mechanisms (Section 4.2.2).

[17] It is interesting that there are few examples of multiple Bayes equilibria in resource allocation models. The apparent example in Laffont and Maskin (1982) is, unfortunately, not valid. See, however, Postlewaite and Schmeidler (1986).

(c) For environments with at least three agents, there exist continuous, balanced, nonparametric, efficient Nash mechanisms (Section 4.1.2).
(d) There do not exist nonparametric, individually feasible, nontrivial, efficient Nash mechanisms (Section 4.1.2).
(e) There exist decentralized, parametric, feasible, efficient Nash mechanisms (Section 4.2.1).
(f) It is an open question whether there exist continuous, parametric, feasible, efficient Nash mechanisms (Section 4.2.1).
(g) There do not exist nonparametric mechanisms whose manipulative Nash allocations are Pareto-efficient (Section 4.1.3).
(h) There exist nonparametric planning procedures that generate efficient allocations under either local maximin behavior or local Nash behavior (Section 4.1.4).
(i) It is an open question whether there exist planning procedures that generate efficient allocations under sequential Bayes behavior (Section 4.1.4).

Finally, we should emphasize the open questions raised by the concept of manipulative equilibrium, by Roberts's work in this volume on planning procedures, and by our justification of the assumption of Nash-equilibrium behavior. There is not a consensus normal- or extensive-form model of rational behavior for agents in iterative processes.

Appendix: Sketches of Proofs

We include in this appendix sketches of proofs of a few of the major theorems of Section 4.

Sketch of Proof of Hurwicz, Maskin, and Postlewaite Theorem 4.15

Hurwicz, Maskin, and Postlewaite (1982) define a decentralizable parametric allocation mechanism that satisfies the conclusion of the theorem in two steps. First, an auxilliary function is given that determines outcomes, given reported endowments w, $h(s_1, \ldots, s_N, w) = a$. This auxilliary function is defined to be feasible, given w, for all messages s, and so that $h(b^N(e;h),w)$ is Pareto-efficient in the environment with preferences as in e and endowments w. Let $s_i = (x_i,p)$ and $S_i = \{(p,x) \in P^1 \times R^1_+ | px = pw_i\}$, where x_i denotes the total consumption (not net trade) of the i^{th} agent.

The outcome function h, for total holdings, is defined as follows:

1. If there exist i, j, $k \in N$ such that p_i, p_j, p_k are distinct, then $h_t(s) = [\| x_t \| / \Sigma_{r \in N} \| x_r \|]w$, for all $t \in N$.
2. If there exist only two distinct announced prices p' and p'', and at least two agents announce each p' and p'', then $h_i(s) = w_i$ for every $i \in N$; that is, there are no trades.

3. If there is a p such that $p_i = p$ for all $i \in N$ (unanimous agreement on the announced price), and

$$\sum_i x_i \neq w, \text{ then } h_i(s) = w_i, \text{ for all } i \in N; \text{ or}$$

$$\sum_i x_i = w, \text{ then } h_i(s) = x_i, \text{ for all } i \in N.$$

4. If there is a p and an agent $m \in N$ such that $p_m \neq p$ but $p_j = p$ for all $j \neq m$, then

$$\begin{cases} h_m(s) = [pw_m/px_m]x_m & \\ & \text{if } [pw_m/px_m]x_m \leq w; \\ h_j(s) = \dfrac{1}{N-1}[w - h_m(s)] \text{ for } j \neq m & \end{cases}$$

$$h_i(s) = w_i, \text{ for all } i \in N, \qquad \text{if } [pw_m/px_m]x_m > w.$$

It can now be shown that $h(b^N(e/w;h),w) = CW(e/w)$ for all $e \in E$ and all $w \geq 0$.

The next step in the proof is to provide a mechanism that yields the correct reported endowment as a Nash equilibrium. Let $m_i = (v_i,x_i,p_i)$, where $x_i \in R^L$, $p_i \in R^L$, and $v_i \in R_+^{NL}$. Note $v_i = (v_{i1},\ldots,v_{iN})$, where v_{ij} can be interpreted as i's report about j's endowment. Let $M_i = \{m_i\}$, $M = \Pi_i M_i$. For all i,j it is required that $v_{ii} \leq w_i$. The outcome function, $h(v,s)$, is defined as follows:

(a) If there is a $\bar{v}$ such that $v_i = \bar{v}$ for all i, then

$$h(m) = h(v,s) \text{ as defined previously.} \tag{1.1}$$

If not, then let

$$v(m) = \sum_i v_{ii}$$

$$A(m) = \{i \in N | v_{ii} \geq v_{ij}, \text{ for all } j \neq i,\ j \in N\},$$

$$B_i(m) = \sum_{j \neq i} \sum_{k \neq i} \| v_{ij} - v_{jk} \|, \text{ for all } i, \text{ and}$$

$$G_i(m) = \sum_j \| v_{ii} - v_{ij} \|, \text{ for all } i.$$

Thus:

(b) If $A(m) = \varnothing$ (the empty set), but there is no $\bar{v}$ such that $v_i = \bar{v}$ for all $i \in N$, then $\Sigma_j B_j > 0$

and we set

$$h_i(m) = \frac{B_i(m)}{\sum_j B_j(m)} v(m) - v_{ii}, \text{ for all } i \in N. \tag{1.2}$$

And

(c) if $A(m) \neq \varnothing$, then $\Sigma_j G_j(m) > 0$

and we set

$$h_i(m) = \begin{cases} \dfrac{G_i(m)}{\sum_j G_j(m)} v(m) - v_{ii} & \text{for } i \in A(m) \\ -v_{ii} & \text{for } i \notin A(m). \end{cases} \tag{1.3}$$

Sketch of Proof of Postlewaite and Wettstein Theorem 4.17

Let $s_i \equiv (p_i,z_i,r_i) \in S_i \times R_+^1$ where $S_i \equiv \{(p_i,z_i) \in R_+^L \times R^L \mid p_i z_i = 0$ and $\Sigma_{k=1}^L p_{ik} = 1\}$. Given messages $s_1,\ldots,s_N$ and endowments $w_1,\ldots,w_N$, define: $a_i \equiv \Sigma_{j,k\neq i}|p_j - p_k|$, $a \equiv \Sigma_{i=1}^N a_i$, $b_i = a_i/a$ if $a > 0$, and $b_i = 1$ if $a = 0$, $\bar{p} = \Sigma_{i=1}^N b_i p_i$. Define x_i to be the closest point to z_i in $\{z \mid p \cdot z = 0, z + w_i \geq 0\}$. Finally, let $h_i'(s_1,\ldots,s_N;w) = r^* \cdot r_i \cdot x_i + (r^* \cdot r_i - 1)w_i$ where $r^* \equiv \max\{r \in R \mid r \cdot r_i \leq 1, \forall\ i, \text{ and } \Sigma_i r \cdot r_i(x_i + w_i) \leq \Sigma_i w_i\}$. Now, in equation (1.1) use $h'(s;w)$ in place of the function $h(s,v)$. The function now defined by (1.1′), (1.2), and (1.3) satisfies the conclusion of the theorem.

Sketch of Proof of D'Aspremont and Gerard-Varet Theorem 4.18

The appropriate mechanism is a Groves mechanism with transfers arranged so that they balance (i.e., the transfers sum to zero). The mechanism is a direct revelation mechanism and chooses $y = y(m_1,\ldots,m_n)$ and $t_i = t_i(m_1,\ldots m_n)$ as follows. Remembering that $u(x_i,y,e_i) = u(y,e_i) + x_i$ for all i, $y(m_1,\ldots,m_n)$ maximizes $W(y,m) = \Sigma_i u(y,m_i)$. Let $x_i(m) = w_i - t_i(m,p)$, where $t_i(m,p) = -\int_{E_{-i}} u(y(m),e_j)dp(e/m_i) + (1/n-1)\,\Sigma_{k\neq i}\int_{E_{-k}} u(y(m),m_j)dp(e/m_k)$. Thus $t_i(m,p) = -g(m_i,p) + (1/n-1)\,\Sigma_{k\neq i} h(m_k,p)$. Given any true value, e_i^*, agent i wants to choose m_i to maximize $u(y(d(e)/m_i),e_i^*) - \int_{E_{-i}} u(y(d(e)/m_i),e_j)\,dp(e/e_i)$. It is easy to verify that $e_i = e_i^*$ solves this problem.

5. Large Economies and Efficient Dominant-strategy Mechanisms

Hurwicz (1972) was well aware that both the pessimistic impossibility results of Section 3 and the need to consider nondominant strategies might disappear if there were a large number of traders. In particular he noted that, regarding incentive compatible and efficient mechanisms, the crucial distinction is whether the economy is atomistic or not. We turn now to an exploration of this observation. The main question of interest is whether some type of approximation to the design of efficient dominant-strategy mechanisms is possible when there are a large number of agents. A second question of interest is whether there is any difference in the answers for private and public goods. The standard approach is first to consider environments with a continuum, or a countable infinity, of agents and then to "pass back" the results, using continuity, to large but finite economies. We follow that approach here.

5.1. Continuum Environments

It has long been conventional wisdom that, in private goods environments, if there are a large number of consumers then price-taking behavior is incentive compatible. We have been unable to find a formal statement and proof of this insight in the literature although there is an implicit understanding of it in a paper by Roberts and Postlewaite (1976). Hammond (1979) presented a theorem by utilizing a model with a continuum of agents that was developed by Aumann (1966). Some ambiguities were later cleared up by Champsaur and Laroque (1981). We present a slightly modified version of this theorem.

THEOREM 5.1. In private goods environments with an atomless continuum of agents, the competitive mechanism defined above in Section 2.3 is an efficient, dominant-strategy mechanism.

PROOF. See Appendix to Section 5 for a sketch of the proof.

This result should surprise no one. The fact that a similar result is obtained in classical public goods environments with a continuum of agents should surprise many since this runs counter to Downs's (1957) and others' intuition. To see why, we first present a specific direct-revelation version of an allocation mechanism for public goods. The message of any agent is that agent's characteristic. The outcome function picks a level of public goods and a vector of net trades in private goods. Given a vector of announced characteristics e', let $P(e')$ be the set of Pareto-efficient allocations for e'. (Remember that these announced characteristics may well be different from the true characteristics making up the true environment, e.) Let $F(e')$ be those allocations in $P(e')$ such that $px_i = pw_i - (1/N)\,qy$, where (p,q) are the prices that support the Pareto-efficient allocation $(x_1,\ldots,x_N,y)$. Then $h(e') = F(e')$ is called the fair-efficient mechanism since all agents "pay" an equal-cost share of the public good. It does not necessarily select Lindahl allocations and is, therefore, different from the privately fair Lindahl mechanism of Hammond (1979), which rarely has an equilibrium. (See Groves and Ledyard 1977, 1980.) Fair-efficient allocations exist unless $(1/N)\,qy > pw_i$ for some i, that is, unless the proportional tax bankrupts some agent. The surprising result is Theorem 5.2.

THEOREM 5.2. In public goods environments with an atomless continuum of agents, the fair-efficient mechanism is an efficient, dominant-strategy mechanism.

PROOF. See Appendix to Section 5 for a sketch of the proof.

It would appear that, in continuum economies as in finite economies, there is fundamentally no difference between private and public goods environments with respect to the possibility of the design of efficient, dominant-strategy mechanisms. However, as noted by Hammond (1979) and Downs (1957), the reason truth is dominant in the fair-efficient mechanism is that changes in d have absolutely no effect on the level of public goods received or on the level of taxes paid. Thus, any d' such that $x(v,d) = x(v,d')$ is a dominant strategy. There is no incentive either to lie or to tell the truth. Muench and Walker (1979) also noted this phenomenon for some versions of the quadratic mechanism. In the private goods case this is not true for the competitive mechanism. Thus there appears to be a subtle difference in the type of result, in spite of the superficial similarities. This difference is most easily highlighted by considering large finite economies.

5.2. Large Finite Economies

To discover what happens in large finite economies, we consider limiting results as the number of agents approaches infinity. We already know that it is impossible to design efficient, dominant-strategy mechanisms in finite economies, even if they are large. However, if there is continuity as the number of agents grows, then the existence of efficient, dominant-strategy mechanisms in large economies should give us some hope that in large finite economies we can have mechanisms that are "almost" efficient, dominant-strategy mechanisms.

5.2.1. Limiting Incentive Compatability

Two papers have addressed this issue by considering the potential gain from misrepresentation. In the first, by Roberts and Postlewaite (1976), a definition of "almost" dominant strategy is given for private goods economies. In particular, they defined a mechanism to be limiting incentive-compatible if for any $\varepsilon > 0$, and any utility function representing an agent's preferences, in sufficiently large economies the gain from using some characteristic other than the truth is less than ε.

Endowing the set of measures that have compact support on D, $\{v\}$, with the topology of weak convergence, we can talk about large finite economies that are "close" to atomless environments. Letting $C(v)$ be the set of competitive equilibrium prices for the environment v, we get the following major result:

THEOREM 5.3 (Roberts and Postlewaite 1976). On the class of classical private goods environments, let $v_k \rightarrow v'$ where v' is an atomless measure. If $C(v)$ is continuous[18] at v', then for any $\varepsilon > 0$ and any utility function there is a k^* such that $k > k^*$ implies that $u(h(v_k),d) > u(h(v_k/d'),d) - \varepsilon$ for any agent d in v_k. (That is, the gains from misrepresentation are arbitrarily small.)

PROOF. See Roberts and Postlewaite (1976).

In the other paper to consider large environments, Roberts (1976) adapted the previous definition of limiting incentive compatibility to public goods environments and looked at the performance of several general classes of mechanisms. We present one of the several impossibility results contained in this paper.

THEOREM 5.4 (Roberts 1976). Let v_k be an expanding sequence of public goods environments (the number of agents increases) with one private good,[19] and let $h(v) = (x(v,d),y(v))$ be an allocation mechanism such that h is individually rational (i.e., $u(h(v),d) \geq u(w(d),0)$), such that $y(v)$ is uniformly continuous on the sequence $\{v_k\}$, and such that $y(v_k) \rightarrow y^*$. If $x(v_k,d) \rightarrow x^* < w$, then h cannot be limiting incentive compatible for the sequence $\{v_k\}$.

PROOF. The proof of this theorem consists of showing that the misrepresentation of acting as if one receives no utility from the public good yields a gain that is bounded away from zero unless the agent's implicit tax goes to zero (for details, see Roberts 1976, p. 367).

This theorem would seem to point to a key difference between private and public goods. However, the fair-efficient mechanism we used earlier in the limit economy is not individually rational and, therefore, is not subject to the conclusion of this theorem. In fact it can be shown that that mechanism is limiting incentive-compatible.

THEOREM 5.5. Consider the class of environments characterized by quasi-linear preferences, one private and one public good, and crowding in the production of the public good so that the optimal quantity of the public good is bounded above by y^* finite. Let v_k be an expanding sequence of public goods environments in this class. There exists an allocation mechanism for this class of environments that is limiting incentive-compatible for the sequence $\{v_k\}$.

PROOF. See Appendix to Section 5 for a sketch of the proof.

[18] For most atomless v', $C(v)$ is continuous at v'.

[19] There does not seem to be anything special about one private good.

It would seem at this point that there is absolutely no difference, from a mechanism design point of view, between public and private goods, since all five of the following facts apply to both classical private goods environments and classical public goods environments.

(1) In finite environments there exists at least one dominant strategy mechanism.
(2) In finite environments there do not exist efficient, dominant-strategy mechanisms if the class of environments is large enough.
(3) In finite environments there exist efficient Nash mechanisms.
(4) In atomless environments there exists at least one efficient, dominant-strategy mechanism.
(5) There exists at least one mechanism that is limiting incentive-compatible, unless individual rationality is required.

What then, if anything, accounts for the conventional wisdom that the incentives in allocating private goods are fundamentally different from those in allocating public goods? Consider an alternative approach.

5.2.2 Nash-Equilibrium Behavior

We have seen[20] that in large finite private economies there are mechanisms such that almost all agents have a small incentive to free ride, when all others act according to their true preferences. It is possible, however, that although each misrepresents only a little, the combined effect is large and the outcome is still inefficient. Therefore, rather than assume that all but one agent behave truthfully, as is done in the definition of limiting incentive-compatibility, let us consider what occurs if all misrepresent. Since there will be no dominant strategies, we will assume Nash-equilibrium behavior.

The intuition that the combined effects can overwhelm limiting incentive-compatibility is definitely correct. This is most easily seen by considering the competitive mechanism in the Edgeworth box environment. We know from the work of Hurwicz (1978), and the extensions of Otanyi and Sicilian (1982), and Thomson (1984) that the set of Nash-equilibrium allocations for the competitive mechanism in the two-person pure exchange economy consists of all allocations in the (lens-shaped) area within the agents' true offer curves. Pick any one of these inefficient allocations and replicate the environment. In each replica, it is possible to construct (misrepresented) offer curves for each agent such that the originally chosen allocation remains a

[20] This section was improved immensely by Andrew Postlewaite, who convinced us that the material in the second paragraph was correct.

Nash-equilibrium allocation. Large numbers seem to eliminate no inefficient alternatives when multiple agents can misrepresent.

On the other hand, it is possible to prove a variation of the Roberts-Postlewaite (1976) theorem and state conditions such that Nash equilibria of the competitive mechanism are almost truthful (and, therefore, almost efficient) in large economies. If one replicates an Edgeworth box environment and selects, for each replica, a Nash equilibrium, and if the misrepresented aggregate demand functions associated with these equilibria converge to an aggregate demand function whose slope is nonzero at the Walrasian-equilibrium price, and if the environments are continuous enough, then it must be true that the Nash-equilibrium misrepresented demand function of each consumer approaches the truthful demand function and, therefore, the Nash-equilibrium allocations approach an efficient allocation. A sketch of the proof is contained in Appendix 5 (Theorem 5.6).

Although the preceding two paragraphs may seem contradictory, the difference is easily explained. For Nash-equilibria preferences to remain misrepresentations as the economy grows, it must be true that the slope of the reported aggregate demand function at the Walrasian-equilibrium price is converging to zero. Only in this situation can each agent have an effect on price by reporting a (false) quantity demanded, since the change in price due to a change in quantity is infinite (in the limit). There are, therefore, two kinds of Nash Equilibria in large economies for the Walrasian mechanism: (1) misrepresented preferences with zero-sloped demand functions yielding inefficient allocations and (2) almost truthful preferences with nonzero-sloped demand functions yielding almost efficient allocations. Fortunately, the former are not very robust, whereas the latter are. To see this intuitively, suppose that each agent may make a very small error in reporting his equilibrium strategy.[21] The reported aggregate demand function will, for most of the aggregate errors, have a nonzero slope at the equilibrium price. Therefore, if the economy is very large, each agent will lose very little by reporting the truth. (It is only if the demand function has a zero slope that the agents could gain by misrepresenting.) For any error that is independent of the size of the economy, there is a large enough economy such that any best replay will be close to the truth. Combining these facts yields Theorem 5.6.

[21]What follows may seem related to concepts of trembling-hand equilibria, but it is not. In each replica, all Nash are also (trembling hand) perfect; standard trembles do not eliminate any of the inefficient equilibria. However, if one chooses the tremble (on a per person basis) independent of the size of the economy, then, for large enough economies, the inefficient equilibria disappear.

THEOREM 5.6. In classical private goods environments, with enough continuity and with some (arbitrarily small) uncertainty in reporting, there is a (direct revelation) mechanism that is almost an efficient dominant-strategy mechanism if the economy is large enough.

PROOF. A mechanism is almost a dominant-strategy mechanism if there is a strategy for each i that is independent of the others' strategy choices and that yields almost as much utility as the best replay. For the competitive mechanism in very large environments this "almost dominant strategy" is the true characteristic. Therefore, the allocations will be almost efficient.

In public goods environments this theorem does not seem to hold. Consider the fair-efficient mechanism presented in Section 5.1. For each agent, d, let $y'(d)$ solve max(*wrt y*) $u(y,d) - f(y)$, and then let d^* solve max(*wrt d*) $y'(d)$. If $y'(d) < y'(d^*)$, then it will be in d's interest to send the misrepresentation, d' where $u(y,d') = 0$ for all y. It will be in d^*'s interest to send the misrepresentation, d'^* where $u(y,d'^*) = u(y,d^*)$ for all y, if all others send d'. Thus the Nash-equilibrium outcome will be $y = y'(d^*)$ and each d will play $f(y'(d^*))$. As the economy grows, these remain the appropriate misrepresentations, and the outcomes and the allocations remain bounded away from efficiency. If we consider the limit of these environments as N grows, we see that although the gain from these "free riding" strategies goes to zero, there is no loss from following them even in the limit. That is, even in the limit environment with an infinite number of agents, these are "reasonable" strategies; nothing is lost by following them; they are optimal. In fact, they are almost dominant. (In the limit economy, since no agent can change either taxes or the level of public good, almost all misrepresentations are as good as the truth.)

The above behavior, as one passes to the continuum, does not occur, however, in all allocation mechanisms for public goods. In a recent paper, Muench (1983, 1) examines the implications of manipulative Nash behavior in large finite economies for the quadratic mechanism of Groves and Ledyard. He shows that, as the environment is replicated, there are local manipulative Nash equilibria of the symmetric Nash equilibrium arbitrarily close to Pareto-efficient allocations and the allocations in the limit involve dividing the cost of the public good equally among all consumers. In our language, the manipulative Nash-equilibrium allocations converge to the fair-efficient allocations as the economy is successively replicated. As we pointed out in Section 4, considering manipulative Nash behavior in a mechanism $h(m)$ is equivalent to considering Nash behavior in the direct revelation mechanism $h[b^N(e;h)] = H(e)$. Thus Muench has, in effect, shown that there is a mechanism, H, whose Nash-equilibrium allocations are approximately efficient in large finite replica economies, with enough continuity.

It would seem that we now have a result for public goods environments like Theorem 5.5 above. But Muench also proves that, in the limit, the Nash equilibria of $H(e)$ involve misrepresentations. This should be evident since the Nash-equilibrium allocations of the quadratic mechanism, h, are not fair, even in the limit. Thus, convergence of the Nash allocations of H to fair efficient allocations implies that the Nash equilibria of H, $b^N(e;H)$, do not converge to e. Therefore, they cannot be approximate dominant strategies.

To summarize, and also to provide a contrast with the private goods environments, we state two propositions.

THEOREM 5.7. In classical public goods environments, (a) there is a (direct revelation) mechanism whose Nash equilibria are almost dominant strategies if the economy is large enough, and (b) there is a (direct revelation) mechanism whose Nash equilibria are almost efficient if the economy is large enough.

PROOF. Summarizes above discussion.

CONJECTURE 5.8. In classical public goods environments, there is no mechanism that, in large economies, is "almost" an efficient, dominant strategy mechanism.

If this conjecture is correct, it is the first fact that differentiates private goods environments from public goods environments.

It should be noted that both direct-revelation mechanisms—the competitive mechanism and the manipulative Nash version of the quadratic mechanism—do not produce efficient allocations in finite environments; only in a limiting sense are they efficient. Yet we know from above that there are other mechanisms that are efficient Nash in all finite environments. Can we find one that, in the limit, is almost an efficient dominant-strategy mechanism? For now our answer is that we do not know. For the mechanisms like those in Section 4, the message space is smaller than the space of classical environments. It thus seems unlikely to us that dominant strategies exist even in the limit.

We can carry this analysis a step further. In private goods economies, the Nash-efficient mechanisms discussed in Section 4 have the additional property that they select Walrasian allocations. That is, $h[b^N(e;h)] \subseteq W(e)$. We know that $W(\cdot)$ is simply the outcome function of the competitive process which is almost a dominant-strategy mechanism in large environments. Therefore, as the private goods environment grows larger, the manipulative Nash equilibria of those Nash-efficient mechanisms converge to a dominant strategy, the true e_i.

COROLLARY 5.9. In large finite private goods environments there are Nash-efficient mechanisms with the property that truth is almost a manipulative Nash equilibrium. That is, it is almost a dominant strategy to employ Nash behavior (according to one's true characteristics) in one's message responses, if everyone else employs Nash behavior (according to some arbitrary characteristic).

A similar result does not seem to be valid in public goods environments. From our work and from that of Muench (1983) and Muench and Walker (1979, 1983), we know that the quadratic mechanism is an efficient Nash mechanism that in the limit is an efficient, but not a dominant-strategy, mechanism. (The manipulative Nash equilibria are not efficient in finite economies but are in the limit.) However, the manipulative Nash equilibria converge neither to the true characteristics nor to a dominant strategy. Thus an analogous result to that of Corollary 5.8 will not hold for the quadratic mechanism. Almost the same conclusions can be reached for any mechanism whose Nash-equilibrium allocations are Lindahl. The only difference is that the manipulative Nash equilibria converge to a dominant strategy (which is to act as if one gets no utility from public goods) but are never efficient even in the limit.

CONJECTURE 5.10. In public goods environments, there are no mechanisms with the property that, in large economies, truthful Nash behavior is almost a dominant strategy, or truth is almost a manipulative Nash equilibrium.

Again we have a subtle but important distinction between private and public goods environments if Conjectures 5.8 and 5.10 can be verified. This work remains to be done.

5.3. Summary

Combining the results in the previous section, we can summarize the state of knowledge concerning the possibilities for the design of efficient, incentive-sensitive mechanisms in "large" economies as follows:

THEOREM 5.11

(a) In classical environments (both public and private) with a continuum of agents, there exist nonparametric, efficient, dominant-strategy mechanisms (Section 5.1).

(b) In classical environments (both public and private), there are mechanisms that are efficient and limiting incentive compatible, if individual rationality is not required[22] (Section 5.2.1).

(c) In classical private goods environments, with enough continuity, there exists a mechanism that is "almost" an efficient, dominant-

[22]This has been proven in the case of environments with public goods only and quasi-linear preferences. The more general statement is conjecture at this point.

strategy mechanism if the economy is "large enough" (Section 5.2.2).

(d) In classical public goods environments, with enough continuity, there exists a mechanism whose Nash-equilibrium allocations are "almost" efficient if the economy is "large enough" and there exists a mechanism whose Nash-equilibrium strategies are "almost" dominant strategies if the economy is "large enough." The two known mechanisms are not the same (Section 5.2.2).

(e) (Conjecture) In classical public goods environments, with enough continuity, there do not exist mechanisms that are "almost" efficient, dominant-strategy mechanisms even in "very large" economies (Section 5.2.2).

(f) In classical private goods environments, there are efficient Nash mechanisms for which truth is "almost" a manipulative Nash equilibrium if the economy is "large enough" (Section 5.2.2).

(g) (Conjecture) In classical public goods environments, there are no efficient Nash mechanisms such that truth is "almost" a manipulative Nash equilibrium even if the economy is "large enough" (Section 5.22).

With these results, we close our survey.

Appendix: 5: Sketches of Proofs

We include in this appendix sketches of proofs of a few of the theorems of Section 5.

Sketch of Proof of Theorem 5.1

The environment is modeled as a measure on a set of possible characteristics. Let D be a set of characteristics—endowments and preferences—and let ν be a measure on that set such that $\nu(D) = 1$. We say that ν is atomless if $\nu(\{d\}) = 0$ for all $d \in D$. [If ν is a finite environment, represented by, say, $e = (e_1, \ldots, e_N)$, then $\nu(\{d\}) = 1/N$ if $d = e_i$ for some i and $\nu(\{d\}) = 0$ otherwise.]

Given an environment ν, let $\nu/(d,d')$ be the same environment with d replaced by d'. If ν is atomless, then for the competitive mechanism the set of equilibrium prices $C(\nu) = C(\nu/(d,d'))$. Thus if some atomless agent reports d' instead of d, there is no effect on the equilibrium price. It follows easily that an agent's best response is the true characteristic no matter what ν is. Thus the competitive mechanism is a truth-dominant mechanism in classical private goods environments with a nonatomistic measure of agents. That the competitive mechanism is efficient is already known.

Sketch of Proof of Theorem 5.2

As in the private goods model, we let ν be the measure on characteristics that describes the environment. The Fair-Efficient mechanism $h(\nu)$ selects a public goods level, $y(\nu)$, and a net trade in private goods for each agent, $x(\nu,d)$, in such a way that there are prices $p(\nu)$ and $q(\nu)$ such that

(1) if $z(\nu) = \int x(\nu,d)d\nu$, then $(y(\nu),z(\nu))$ solves $\max p(\nu)z + q(\nu)y$ subject to $(z,y) \in Y$,

(2) $x(\nu,d)$ solves $\max u(y(\nu),x;d)$ subject to

$$p(\nu)x + \frac{1}{N} q(\nu)y(\nu) \leq p(\nu)w(d) + m(\nu,d), \text{ where}$$

$$\int m(\nu,d)d\nu = p(\nu)z(\nu) + q(\nu)y(\nu), \text{ and } \int d\nu = N,$$

(3) $$\int \left\{ \frac{\partial u[z(\nu),w + x(\nu,d),d]/\partial y}{\partial u[z(\nu),w + x(\nu,d),d]/\partial x_1} \right\} d\nu = q(\nu).$$

It remains to be shown that this mechanism, $h(\nu)$, is an efficient, dominant-strategy mechanism if ν is atomless. It is obviously efficient if truth is a dominant strategy. To see that it is a dominant-strategy mechanism, consider how each of the above three relations change as one atomless agent replaces d with d'. First, none of $z(\nu)$, $p(\nu)$, $q(\nu)$, or $y(\nu)$ change. Thus the only change that the agent can effect is in $h(\nu,d)$, but it is then optimal to send the true d'. Therefore, sending the true d is a dominant response.

Sketch of Proof of Theorem 5.5

To understand the proof, consider a simplified set of environments: those with quasi-linear prefences, one private and one public good, and crowding in production so that the optimal quantity of the public good y_k converges to a finite y^* as the number of agents k becomes large.

Let N be the number of agents in the economy. Assume that $g(y)$ is the amount of private good needed to produce y, with $g(y) = Nf(y)$, and $g'(y) = Nf'(y)$. In this case the fair-efficient mechanism introduced earlier is given by: $x(\nu,d) = -(1/N)\, g(y(\nu)) = -f(y)$, and $y(\nu)$ solves

$$\int \frac{\partial u(y,d)}{\partial y} d\nu = g'(y) = Nf'(y). \tag{*}$$

To see that this is indeed limiting incentive compatible, assume that ν is finite and consider an agent's decision as to which characteristic to report. If d reports d', then $y = y(\nu/d')$ and $x = x(\nu/d',d') = -(1/N)\, g(y(\nu/d')) = -f'(y(\nu/d'))$. Therefore, this agent should *select* his best y' and then choose d' such that $y(\nu/d') = y'$. By *best* we mean that y' which maximizes $u(y,d) - f'(y)$. As N becomes large y' does not change, because of the crowding assumption. Given y', let us see how to calculate d'. Remember that $y(v)$ solves (*). Suppose that $y' < y(v)$ and that one cannot claim that y is a public bad. Then the most we can gain by misrepresenting is by sending d' such that $u(y,d') = 0$ for all y; that is, to claim to have no interest in the public good. The new public good level $y(v/d')$ solves $\int \partial u(y,d)/\partial y \, dv - v(d)\, \partial u(y,d)/\partial y = g(y)$. It is easy to see that as N grows $y(v/d') \rightarrow y(v)$, since $v(d) \rightarrow 0$. It follows that the incentive to misrepresent, $u(y(v/d'),d) - u(y(v),d) - [f(y(v/d')) - f(y(v))]$, goes to 0. Basically, as N grows large, all agents' ability to manipulate y grows small and therefore the gain grows small. Thus, this mechanism is limiting incentive compatible.

Sketch of Proof of Theorem 5.6

Let $x(p,e)$ solve max $u(x,e)$ subject to $px = pw$. (Assume $w_1, \ldots w_N$ are known.) Let $e^* = (e_1^*, \ldots, e_N^*)$ denote true preferences. Let $E = E_1 \times \ldots \times E_N$ be such that (1) $u \in C^4$, (2) there exists a unique competitive equilibrium price system, and (3) $\partial \Sigma x(p,e_i)/\partial p < 0$ at that equilibrium. At the k^{th} replicate misrepresentation Nash equilibrium,

(1) e^k is the equilibrium where if two agents, i and j, are the same type, then $e^{k_i} = e^{k_j}$.
(2) p^k solves $\Sigma x(p,e^{k_i}) - w_i = 0$.
(3) for all i, $\nabla_x u(v,e_i^*)[k \,\Sigma_h x_{hp}(p^k,e^{k_h}) - x_{ip}(p^k,e^{k_i})]\, p^k_{e_i} = 0$

where

$$v = -k \sum_h x_h(p^k,e^{k_h}) + x_i(p^k,e^{k_i}) = x_i(p^k,e^{k_i}).$$

From (2),

$$p_{e_i} = \frac{-\partial x_i(p^k,e^{k_i})/\partial e_i}{k \sum_h \dfrac{\partial x_h(p^k,e^{k_h})}{\partial p}}.$$

Let

$$e^k \rightarrow \hat{e} \text{ where } \hat{e} \neq e^* = \text{(true characteristic) and } p^k \rightarrow \hat{p}.$$

From (3),

$$\nabla_x u(v^k,e_i^*)\left[\frac{\partial x_i}{\partial e_i} - \frac{\partial x_i}{\partial p}\cdot\frac{\partial p^k}{\partial e_i}\right] = 0.$$

Therefore, if

$$\lim_{k\rightarrow\infty} \nabla_x u(v^k,e_i^*)\left[\frac{\partial x_i}{\partial e_i} - \frac{\partial x_i}{\partial p}\cdot\frac{\partial p^k}{\partial e_i}\right] = 0,$$

then

$$\nabla_x u(x_i(\hat{p},\hat{e}),e_i^*) \times \frac{\partial x_i(\hat{p},\hat{e})}{\partial e_i} = 0.$$

But this will be true if and only if $x(\hat{p},e_i^*) = x(\hat{p},\hat{e}_i)$ for all i. Therefore, $\hat{p} = p^*$ and $x_i = x_i(p^*,e_i^*)$.

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